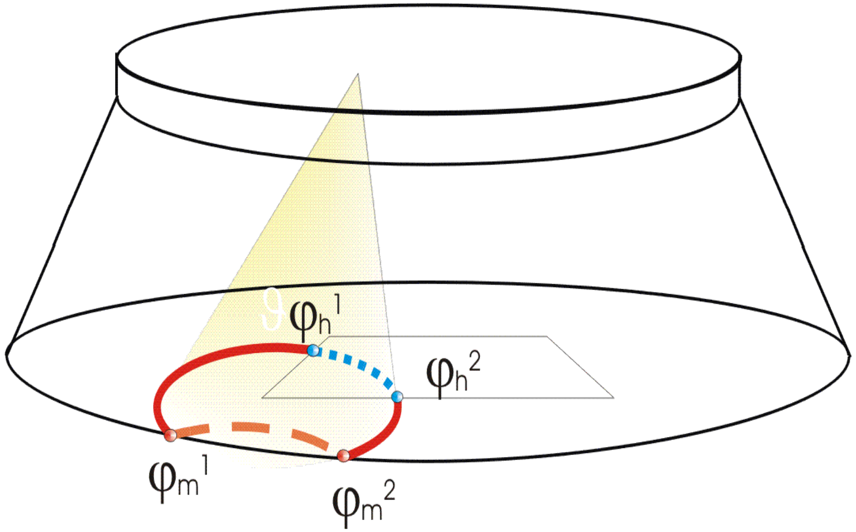
Geometrical Acceptance of the Cherenkov Pattern



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Outlook

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Photon Pattern Tracing

When a charged particle crosses the radiator with a speed greater than c_{medium}, a certain number of photons is emitted isotropically around the particle with an aperture angle q_c

The photons interact through:

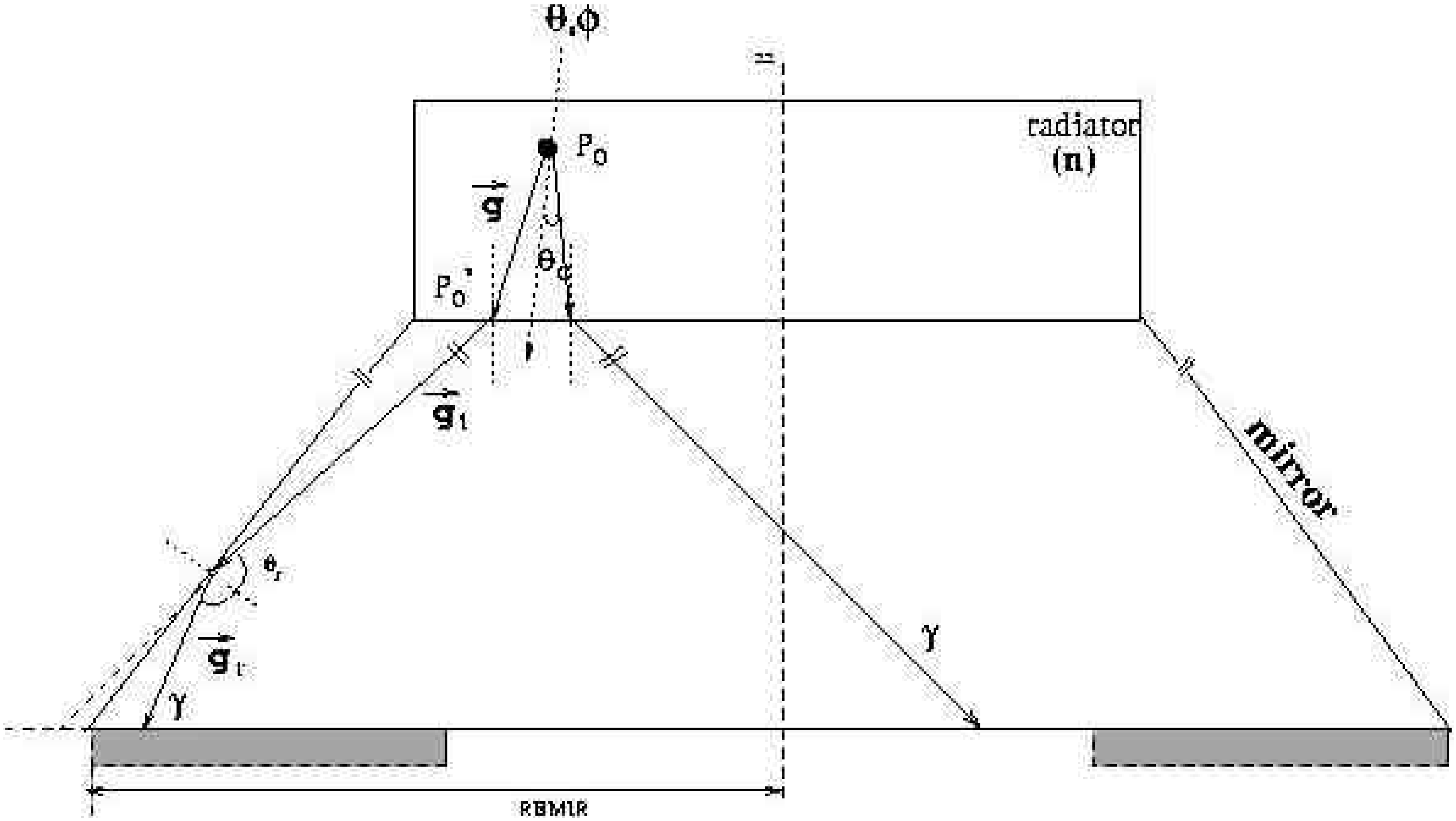
- refraction on radiator boundary (and eventually total reflection, depending on their incident angle q_i)

or

- reflection on conical mirror

A hit pattern is produced in PMT matrix with a Geometrical Acceptance depending on the radiator's impact point P, particles direction (q,f) and Cherenkov angle q_c.

Photon Pattern Tracing

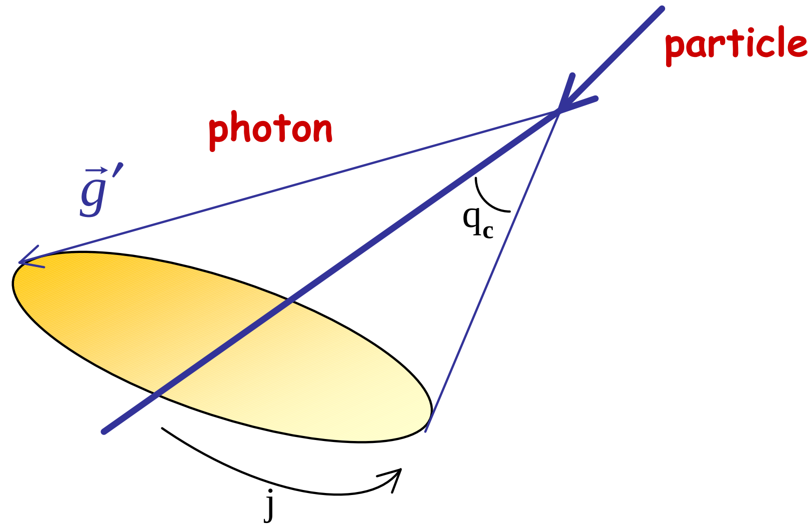


Geometrical Calculations

Particle´s frame:

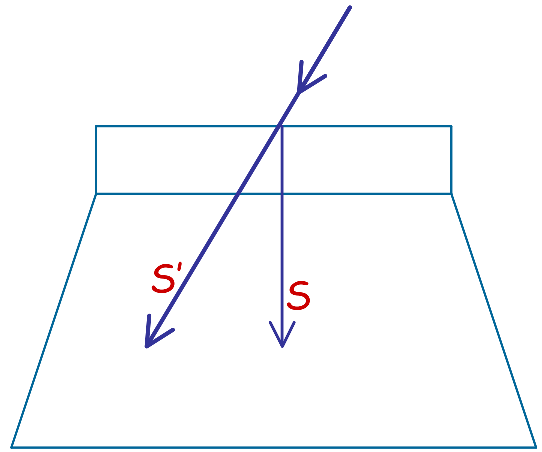
The parametric equations of the photon in terms of the azimutal angle j are:

$$\vec{g}\,'(q_c, j) = (\sin q_c \cos j, \sin q_c \sin j, \cos q_c)$$



Geometrical Calculations

Particle´s frame/Detector´s frame



Transformation matrix elements: $T_{i,j} = \vec{e}_i \cdot \vec{e}_{j'}$

$\vec{e}_i, \vec{e}_{j'}$ **: directions of the two frames axis**

S´ particle's frame

S detector's frame

$$\vec{e}_x = (1,0,0) \qquad \vec{e}_{z'} = (\sin q \cos f, \sin q \sin f, \cos q)$$

$$\vec{e}_y = (0,1,0) \qquad \vec{e}_{y'} = (\sin f, -\cos f, 0)$$

$$\vec{e}_z = (0,0,1) \qquad \vec{e}_{x'} = \vec{e}_{y'} \times \vec{e}_{z'} = (-\cos q \cos f, -\cos q \sin f, \sin q)$$

Geometrical Calculations

The photon's direction is tranformed and in the detector's comes:

$$\vec{\gamma} \equiv \begin{pmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \end{pmatrix} = \underbrace{\begin{pmatrix} -\cos\theta\cos\phi & \sin\phi & \sin\theta\cos\phi \\ -\cos\theta\sin\phi & -\cos\phi & \sin\theta\sin\phi \\ \sin\theta & 0 & \cos\theta \end{pmatrix}}_{T_{i,j}} \cdot \begin{pmatrix} \gamma'_x \\ \gamma'_y \\ \gamma'_z \end{pmatrix}$$

Intersection point with radiator side boundaries:

- This is solution of the following equation:

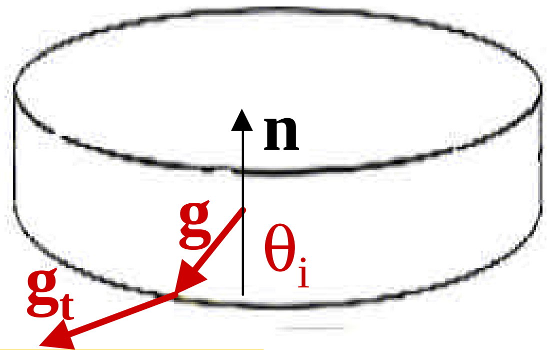
$$(z-z_0)^2\left[\left(\frac{g_x}{g_z}\right)^2+\left(\frac{g_y}{g_z}\right)^2\right]+2(z-z_0)\left[x_0\frac{g_x}{g_z}+y_0\frac{g_y}{g_z}\right]+\left(x_0{}^2+y_0{}^2-RTMIR^2\right)=0$$

Geometrical Calculations

Photon Refraction at radiator boundary:

According to the transformation the incident angle, θ_i, in the radiator´s boundary is obtained as:

$\cos \theta_i = \sin\theta \sin\theta_c \cos\varphi + \cos\theta \cos\theta_c$



- $n \sin \theta_i > 1$ → total reflection
- $n \sin \theta_i < 1$ → refraction in point

$$P'(x'_0, y'_0, z'_0) = P_0(x_0, y_0, z_0) + \frac{(z'_0 - z_0)}{g_z}\vec{g}$$

$z'_0 \equiv$ radiator basis plane

Transmition direction:

$$\vec{g}_t = \frac{\sin\theta_t}{\sin\theta_i}\vec{g} + \frac{\sin(\theta_t - \theta_i)}{\sin\theta_i}\vec{n} = n\vec{g} + \frac{\sin(\theta_t - \theta_i)}{\sin\theta_i}\vec{n}$$

$$\vec{n} \equiv (0,0,-1)$$

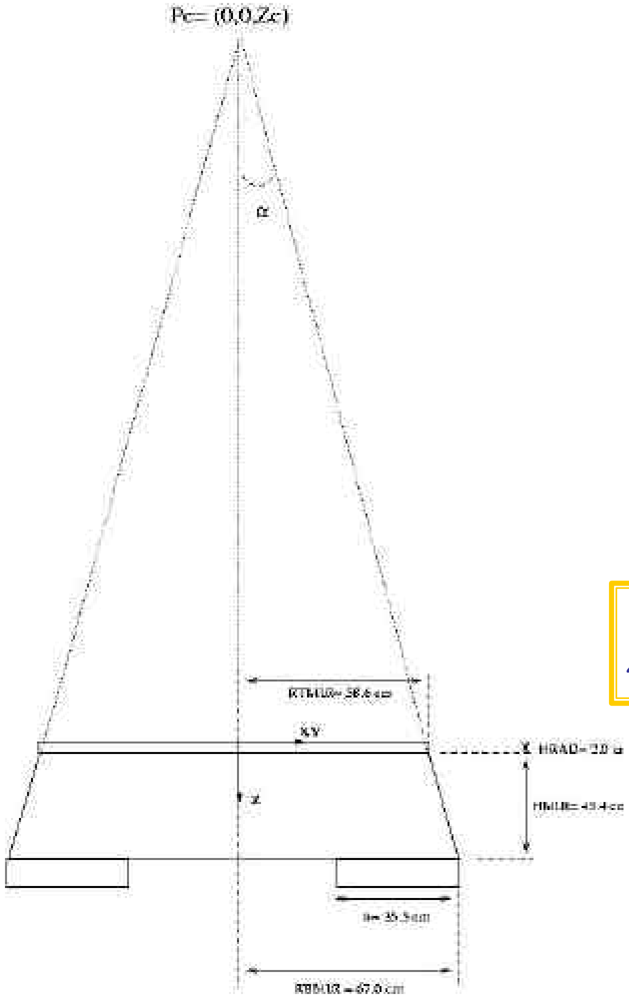
Where:

θ_i incident angle measured with respect to the normal

θ_t transmition angle obtained from Snell's law: $\sin \theta_i = n \sin \theta_t$

Geometrical Calculations

Mirror Surface:



$$\tan a \equiv \frac{RBMIR - RTMIR}{HMIR}$$

$$z_c \equiv \frac{RTMIR \times HMIR}{RBMIR - RTMIR} - HRAD$$

$$x^2 + y^2 - \tan a^2 (z - zc)^2 = 0$$

Intersection point of the photon with the mirror:

- The z-coordinate of the intersection point is solution of the equation:

$$(z-z'_0)^2\left[\left(\frac{g_{tx}}{g_{tz}}\right)^2+\left(\frac{g_{ty}}{g_{tz}}\right)^2-\tan q_c{}^2\right]+2(z-z'_0)\left[\frac{g_{tx}}{g_{tz}}(x'_0-x_c)+\frac{g_{ty}}{g_{tz}}(y'_0-y_c)-\tan q_c{}^2(z'_0-z_c)\right]+(x'_0-x_c)^2+(y'_0-y_c)^2$$
$$-\tan q_c{}^2(z'_0-z_c)^2=0$$

- The x, y coordinates are obtained from:

$$x = x'_0 + \left(\frac{g_{tx}}{g_{tz}}\right)(z - z'_0)$$
$$y = y'_0 + \left(\frac{g_{ty}}{g_{tz}}\right)(z - z'_0)$$

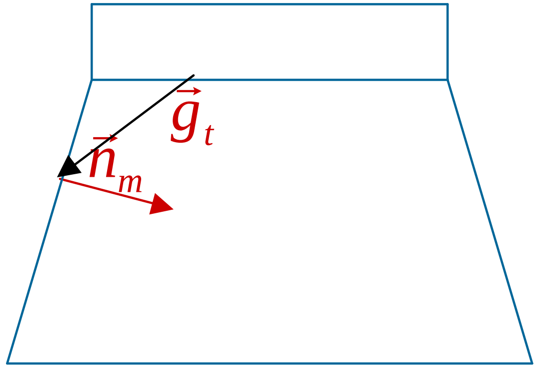
Geometrical Calculations

Photon reflection on mirror:

The reflected photon's direction can be expressed in terms of:

- the normal to the mirror's wall $\vec{n}_m$ (gradient of the conical surface, pointing inward)
- new incident direction $\vec{g}_t$

$$\vec{g}_r = \vec{g}_t + 2\cos\theta_i \vec{n}_m$$



Photon reflection will only happen for those photon's which intersept the PMTs' plane (P_I) with a distance to the z-axis greater than RBMIR

This distance can be computed from the coordinates of the intersection point in radiator and taking into account the height of the mirror (ΔZ_{MIR}).

$$P_I = P_0' + \frac{\Delta Z_{MIR}}{g_{tz}} \vec{g}_t$$

Geometrical Calculations

At this stage photon's will either:

- Reach the PMTs matrix
- Fall in the squared hole on the top of the EMC

Intersection point with the limits of the squared hole:

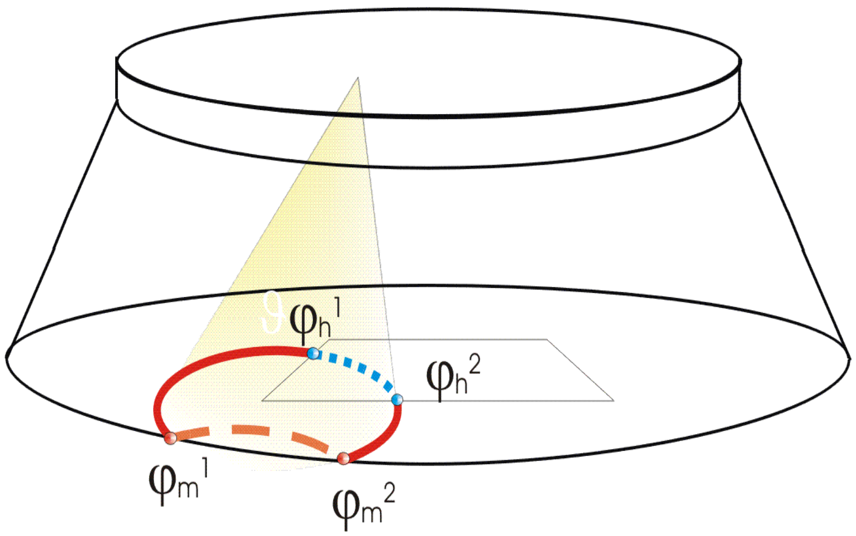
$$\pm x_{\lim} = x_r + \frac{g_{rx}}{g_{rz}}\left(z_{plane} - z_{I0}\right) \qquad \pm x_{\lim} = x'_0 + \frac{g_{tx}}{g_{tz}}\left(z_{plane} - z'_0\right)$$

or

$$\pm y_{\lim} = y_r + \frac{g_{ry}}{g_{rz}}\left(z_{plane} - z_{I0}\right) \qquad \pm y_{\lim} = y'_0 + \frac{g_{ty}}{g_{tz}}\left(z_{plane} - z'_0\right)$$

Acceptances Calculation

Achieved the intersection point the acceptance calculus of each portion in the different detector elements is imediate, once <u>in the particle's frame the photon distribution is uniform!!!</u>



Direct pattern:

$$Dir\_Acc = \frac{(j_h^1 - j_m^1) + (j_m^2 - j_h^2)}{Dj_{TOT}}$$

Pattern in the Mirror :

$$Mir\_Acc = \frac{(j_m^2 - j_m^1)}{Dj_{TOT}}$$

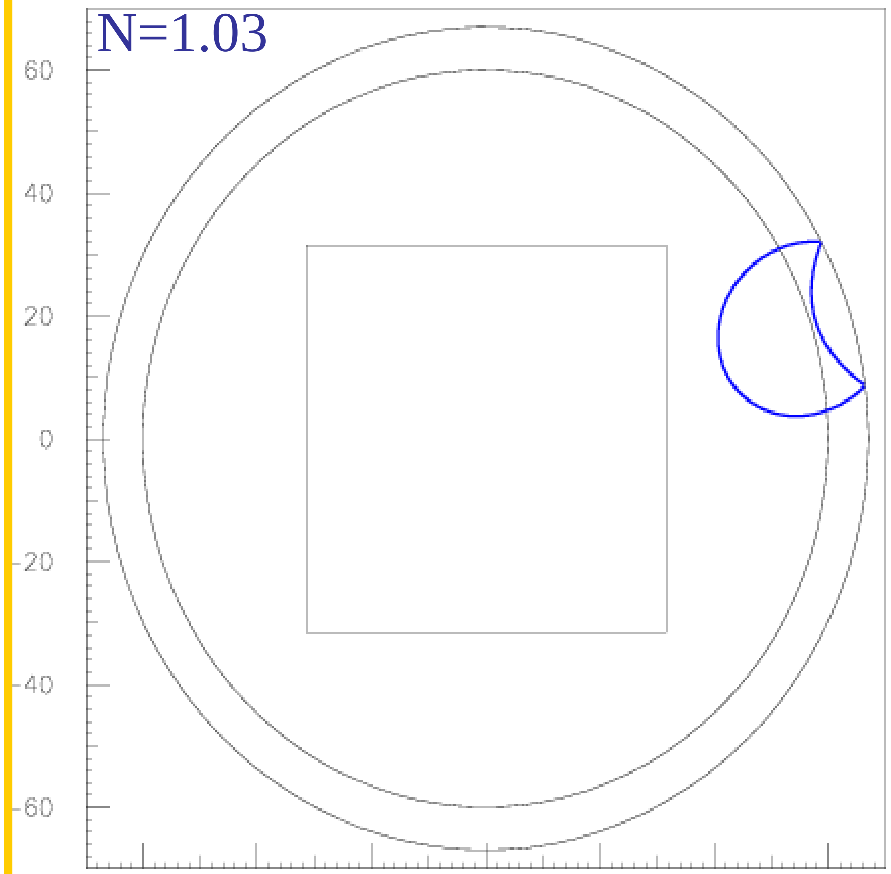
Pattern in the Hole :

$$Hol\_Acc = \frac{(j_h^2 - j_h^1)}{Dj_{TOT}}$$

Geometrical Acceptance:

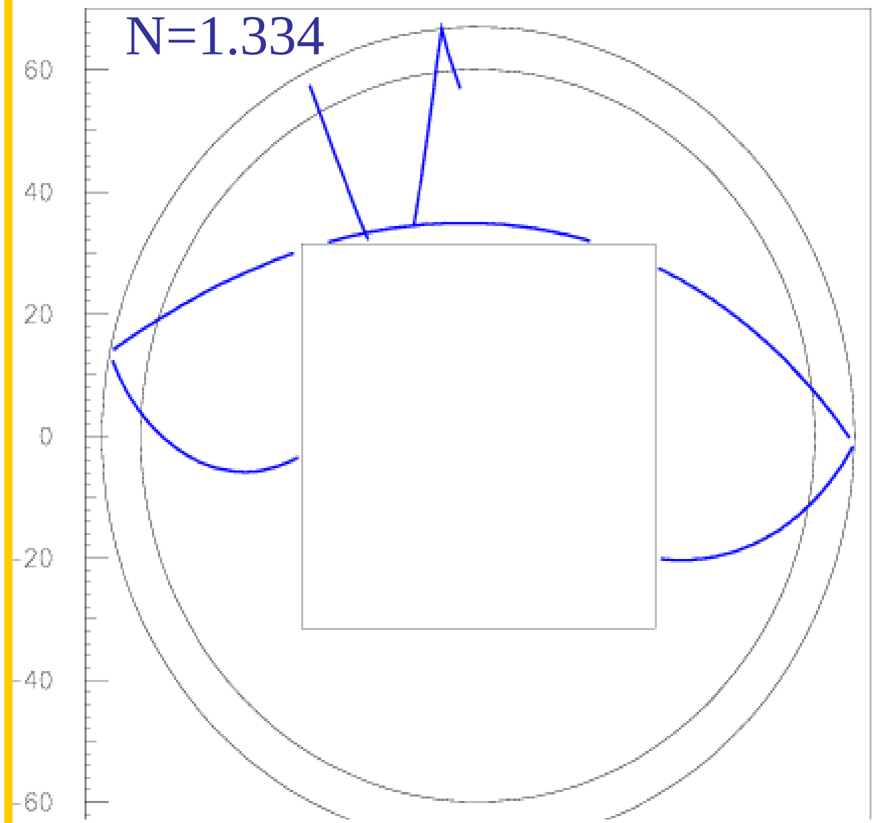
Dir_Acc + Mir_reflectivity * Mir_Acc

Events and corresponding acceptances produced in AGL and NaF



ImpPoint (32.12, 5.25, 0.00) θ = 25.8 ϕ = 27.9

DIRAcc=68.5% MIRAcc=31.5% HOLAcc=0% INVAcc=0%



ImpPoint (9.21,- 17.28, 0.00) θ = 8.3 ϕ = 249.8

DIRAcc=30..9% MIRAcc0=26..3% MIRAcc1=0.5%
HOLAcc=0% INVAcc=42.3%

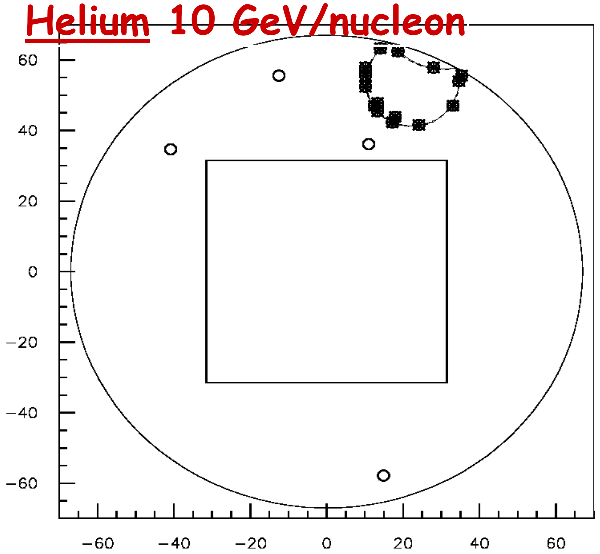
Results

Reconstructed patterns

Impact point (28,-33,0)
q = 12°
f = 290°
b= 0.996

No of: hits = 6
used hits = 4
photoelectrons = 4

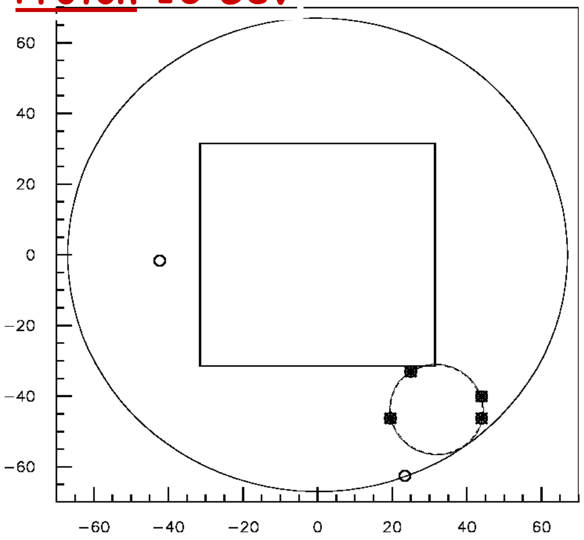
Helium 10 GeV/nucleon



Impact point (20,20,0)
q = 15°
f = 80°
b= 0.995

No of: hits = 33
used hits = 25
photoelectrons = 32

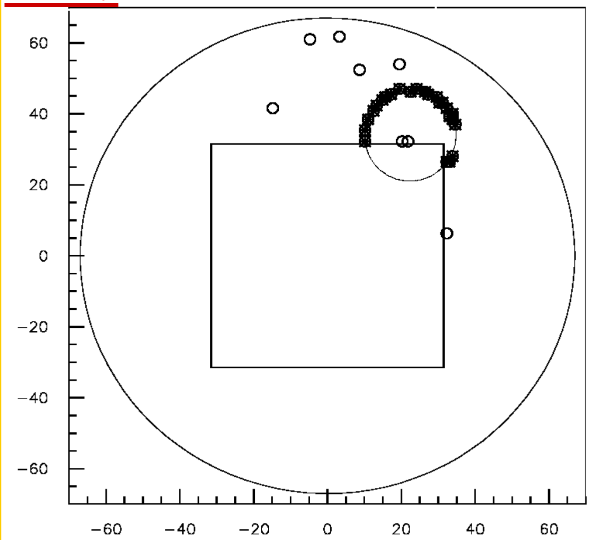
Proton 10 GeV



Impact point (20,40,0)
q = 15°
f = 80°
b= 0.995

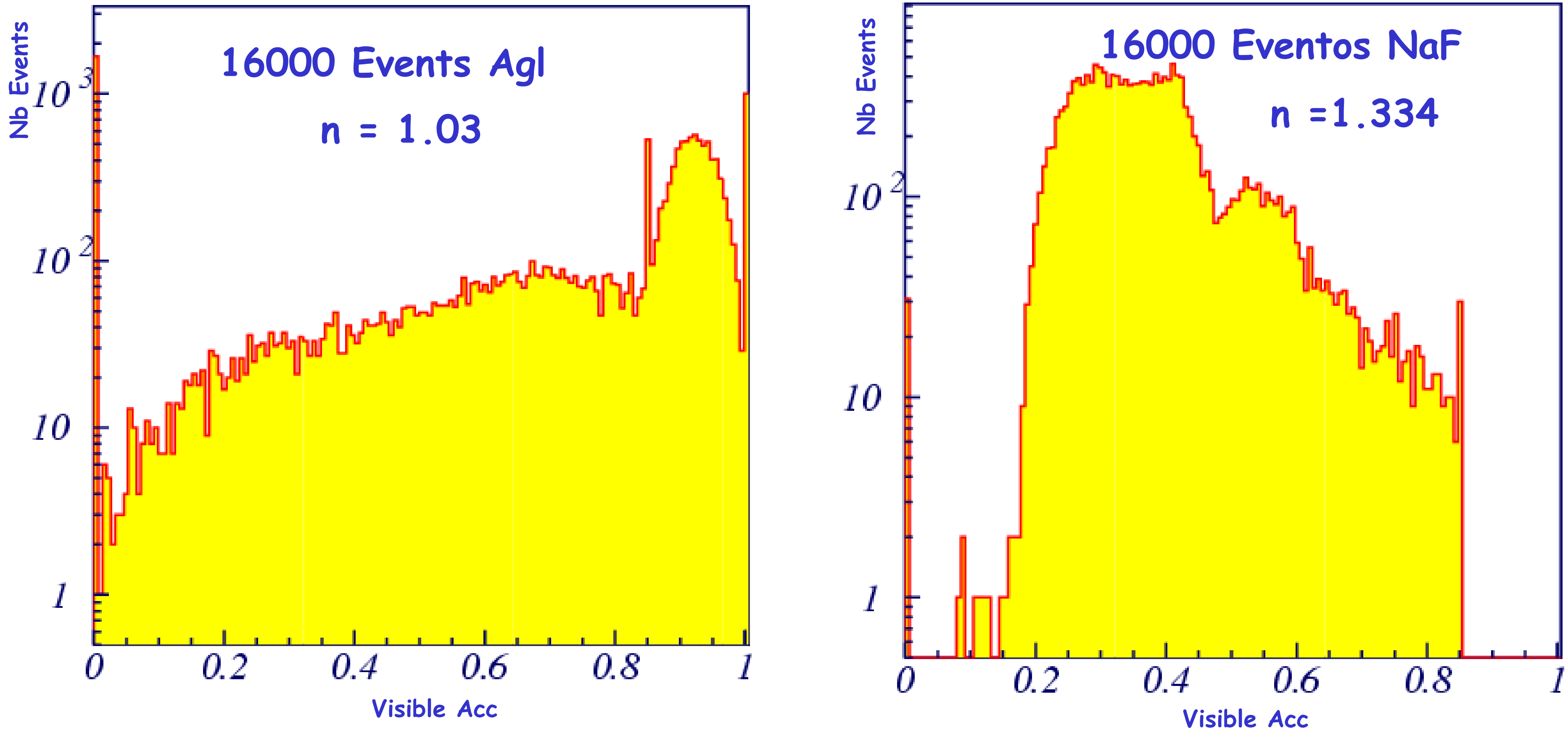
No of: hits = 19
used hits = 15
photoelectrons = 15

Litium 10 GeV/nucleon



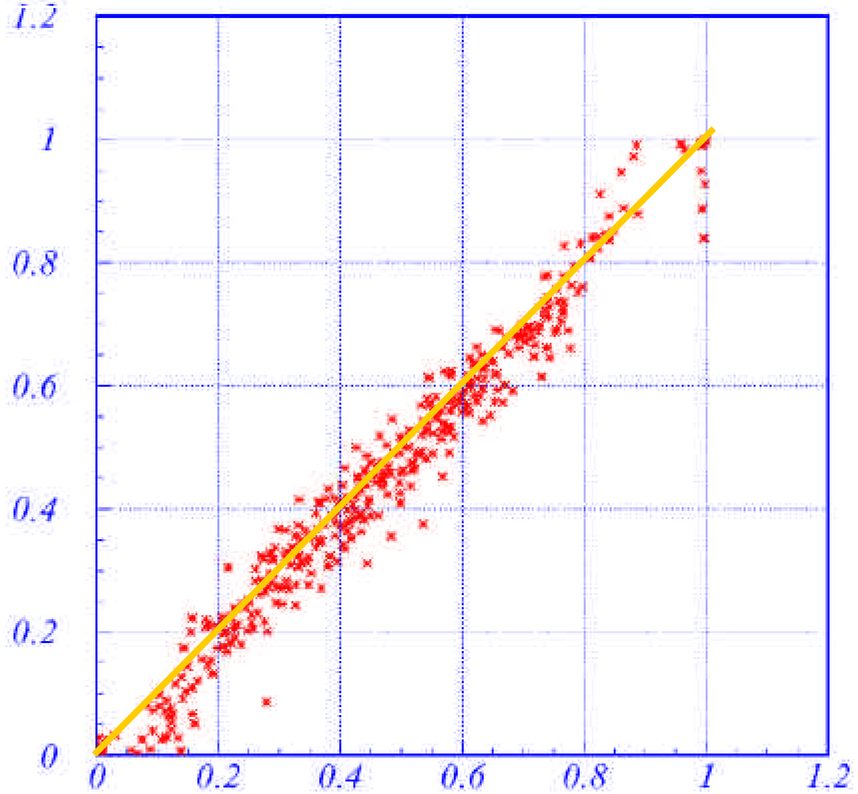
Results

Acceptances distributions



Results

Simulation



Analytical calculation

Simulation

$$\frac{\text{No. of reflected photons}}{\text{No. of observed photons}}$$

Analytical Calculation

$$\frac{0.9 * Mir\_Acc}{Dir\_Acc + 0.9 * Mir\_Acc}$$

Results

This study is applied to:

- ✓ Velocity reconstruction
- ✓ Charge reconstruction
- ✓ Evaluation of eventual inefficient regions of the detector
- ✓ Study of the radiator light yield mapping