Exercises

The Experiment

Consider a WIMP search experiment based on a 2-phase xenon TPC. As other detectors based in this technology, the experiment measures the S1 (scintillation) and S2 (ionisation) signals for each interaction. So it can do 3D position reconstruction of each event, and define a "fiducial" volume where the background is minimal. It can also do NR vs ER discrimination (based on the S2/S1 ratio).
Assume that:

- The fiducial mass is 5600 kg
- The acceptance for nuclear recoils is 50%
- The experiment runs for 3 years with an average 90% duty cycle

This experiment is searching for nuclear recoils caused by WIMPs in the energy range between 5 and 50 keV.

We will do a simplified analysis, using the "cut-and-count" method.

After a thorough analysis of all the background sources (ER and NR), and defining the WIMP search region, we expect 1.5 events in our search region (before unblinding the science data). After unblinding, 4 events are observed!

Exercise 1: Have we found dark matter?!
Find the 90% confidence level interval on the mean number of signal counts (μ) from Feldman-Cousins statistics (see Table IV). Would you claim a WIMP discovery with this experiment? Justify.

Exercise 2: WIMP-nucleus cross section limit
Estimate the experiment sensitivity for a 50 GeV WIMP. Start from the differential nuclear recoil spectrum we expect from WIMPs (slide 18 in Lesson 2):

$$\frac{dR}{dE_R} = \frac{\rho_0 \sigma_A}{2 m_\chi \mu_A^2} F^2(q) \int_{v_{\min}}^{v_{\max}} \frac{f(\vec{v})}{v} d^3 v$$

dR is the NR differential rate (in ev/kg/day/keV), dE_R is our energy range, ρ_0 is the local WIMP density, σ_A is the WIMP-nucleon cross section, m_χ is the WIMP mass, μ_A is the reduced WIMP-nucleus system mass, *F(q)* is the nuclear form factor and *f(v)* is the WIMP Maxwellian velocity distribution.

For simplicity, let's assume:

- The Earth is stationary
- There is no upper limit on the velocity of the DM particles
- The form factor is 1

With these simplifications, this recoil rate can be well approximated by (see Eq. 3.10 in the Lewin&Smith paper in the bibliography)

$$\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-\frac{E_R}{E_0 r}}$$

R_0 is a reference integral rate (normalisation), E_0 is the most probable WIMP kinetic energy and r is the kinematic factor:

$$R_0 = \frac{2\rho_0 \sigma_A v_0}{m_\chi m_A \sqrt{\pi}} \qquad r = \frac{4 m_\chi m_N}{(m_\chi + m_N)^2} \qquad f(\boldsymbol{v}, \boldsymbol{v_E}) = A e^{-(\boldsymbol{v}+\boldsymbol{v_E})^2/v_0^2}$$

Exercise 3: WIMP-nucleon cross section limit
Use the WIMP-nucleus cross section obtained in the previous exercise to calculate the cross section per nucleon. Use the equation from slide 22 in Lesson 2 (or the L&S paper).

$$\sigma_A^{SI}(q \to 0) = \frac{4\mu_A^2}{\pi}[Zf_p + (A-Z)f_n]^2 \approx \frac{\mu_A^2}{\mu_p^2}\sigma_p A^2$$

Exercise 4: Minimum WIMP mass
What is the minimum WIMP mass our detector is sensitive to?
For this exercise, use the standard Earth velocity (v_E = 220 km/s) and WIMP escape velocity from the Milky Way halo (v_{esc} = 544 km/s). The recoil energy is related with the initial WIMP energy (E_i) by the simple kinematic relation (θ is the scattering angle):

$$E_R = E_i r \frac{(1-\cos\theta)}{2}$$

Homework: Sensitivity curve

Generalize the calculations from exercises 2 and 3 to create a plot of the experiment sensitivity (at 90%, 95% or 99% CL) to the SI WIMP-nucleon interaction cross section as a function of the WIMP mass (1 GeV to 1 TeV). Feel free to use your favourite tool (Python, GNUPlot, Excel, Google Sheets, etc). You can even do it manually!

<u>Useful quantities:</u>
220 km/s = 0.75 x 10^{-3} c
1 GeV/c^2 = 1.78266 x 10^{-27} kg
m_{Xe} = 122.3 GeV/c^2
m_p = 0.938 GeV/c^2
A_{Xe} = 131.2

From: G.J. Feldman, R.D. Cousins, “A Unified Approach to the Classical Statistical Analysis of Small Signals”, PRD 57 (1998) 3873, arXiv:physics/9711021

TABLE IV. 90% C.L. intervals for the Poisson signal mean μ, for total events observed n_0, for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 2.44	0.00, 1.94	0.00, 1.61	0.00, 1.33	0.00, 1.26	0.00, 1.18	0.00, 1.08	0.00, 1.06	0.00, 1.01	*0.00, 0.98*
1	0.11, 4.36	0.00, 3.86	0.00, 3.36	0.00, 2.91	0.00, 2.53	0.00, 2.19	0.00, 1.88	0.00, 1.59	0.00, 1.39	0.00, 1.22
2	0.53, 5.91	0.03, 5.41	0.00, 4.91	0.00, 4.41	0.00, 3.91	0.00, 3.45	0.00, 3.04	0.00, 2.67	0.00, 2.33	0.00, 1.73
3	1.10, 7.42	0.60, 6.92	0.10, 6.42	0.00, 5.92	0.00, 5.42	0.00, 4.92	0.00, 4.42	0.00, 3.95	0.00, 3.53	0.00, 2.78
4	1.47, 8.60	1.17, 8.10	0.74, 7.60	0.24, 7.10	0.00, 6.60	0.00, 6.10	0.00, 5.60	0.00, 5.10	0.00, 4.60	0.00, 3.60
5	1.84, 9.99	1.53, 9.49	1.25, 8.99	0.93, 8.49	0.43, 7.99	0.00, 7.49	0.00, 6.99	0.00, 6.49	0.00, 5.99	0.00, 4.99
6	2.21,11.47	1.90,10.97	1.61,10.47	1.33, 9.97	1.08, 9.47	0.65, 8.97	0.15, 8.47	0.00, 7.97	0.00, 7.47	0.00, 6.47
7	3.56,12.53	3.06,12.03	2.56,11.53	2.09,11.03	1.59,10.53	1.18,10.03	0.89, 9.53	0.39, 9.03	0.00, 8.53	0.00, 7.53
8	3.96,13.99	3.46,13.49	2.96,12.99	2.51,12.49	2.14,11.99	1.81,11.49	1.51,10.99	1.06,10.49	0.66, 9.99	0.00, 8.99
9	4.36,15.30	3.86,14.80	3.36,14.30	2.91,13.80	2.53,13.30	2.19,12.80	1.88,12.30	1.59,11.80	1.33,11.30	0.43,10.30
10	5.50,16.50	5.00,16.00	4.50,15.50	4.00,15.00	3.50,14.50	3.04,14.00	2.63,13.50	2.27,13.00	1.94,12.50	1.19,11.50
11	5.91,17.81	5.41,17.31	4.91,16.81	4.41,16.31	3.91,15.81	3.45,15.31	3.04,14.81	2.67,14.31	2.33,13.81	1.73,12.81
12	7.01,19.00	6.51,18.50	6.01,18.00	5.51,17.50	5.01,17.00	4.51,16.50	4.01,16.00	3.54,15.50	3.12,15.00	2.38,14.00
13	7.42,20.05	6.92,19.55	6.42,19.05	5.92,18.55	5.42,18.05	4.92,17.55	4.42,17.05	3.95,16.55	3.53,16.05	2.78,15.05
14	8.50,21.50	8.00,21.00	7.50,20.50	7.00,20.00	6.50,19.50	6.00,19.00	5.50,18.50	5.00,18.00	4.50,17.50	3.59,16.50
15	9.48,22.52	8.98,22.02	8.48,21.52	7.98,21.02	7.48,20.52	6.98,20.02	6.48,19.52	5.98,19.02	5.48,18.52	4.48,17.52
16	9.99,23.99	9.49,23.49	8.99,22.99	8.49,22.49	7.99,21.99	7.49,21.49	6.99,20.99	6.49,20.49	5.99,19.99	4.99,18.99
17	11.04,25.02	10.54,24.52	10.04,24.02	9.54,23.52	9.04,23.02	8.54,22.52	8.04,22.02	7.54,21.52	7.04,21.02	6.04,20.02
18	11.47,26.16	10.97,25.66	10.47,25.16	9.97,24.66	9.47,24.16	8.97,23.66	8.47,23.16	7.97,22.66	7.47,22.16	6.47,21.16
19	12.51,27.51	12.01,27.01	11.51,26.51	11.01,26.01	10.51,25.51	10.01,25.01	9.51,24.51	9.01,24.01	8.51,23.51	7.51,22.51
20	13.55,28.52	13.05,28.02	12.55,27.52	12.05,27.02	11.55,26.52	11.05,26.02	10.55,25.52	10.05,25.02	9.55,24.52	8.55,23.52

TABLE VI. 95% C.L. intervals for the Poisson signal mean μ, for total events observed n_0, for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 3.09	0.00, 2.63	0.00, 2.33	0.00, 2.05	0.00, 1.78	0.00, 1.78	0.00, 1.63	0.00, 1.63	0.00, 1.57	*0.00, 1.54*
1	0.05, 5.14	0.00, 4.64	0.00, 4.14	0.00, 3.69	0.00, 3.30	0.00, 2.95	0.00, 2.63	0.00, 2.33	0.00, 2.08	0.00, 1.88
2	0.36, 6.72	0.00, 6.22	0.00, 5.72	0.00, 5.22	0.00, 4.72	0.00, 4.25	0.00, 3.84	0.00, 3.46	0.00, 3.11	0.00, 2.49
3	0.82, 8.25	0.32, 7.75	0.00, 7.25	0.00, 6.75	0.00, 6.25	0.00, 5.75	0.00, 5.25	0.00, 4.78	0.00, 4.35	0.00, 3.58
4	1.37, 9.76	0.87, 9.26	0.37, 8.76	0.00, 8.26	0.00, 7.76	0.00, 7.26	0.00, 6.76	0.00, 6.26	0.00, 5.76	0.00, 4.84
5	1.84,11.26	1.47,10.76	0.97,10.26	0.47, 9.76	0.00, 9.26	0.00, 8.76	0.00, 8.26	0.00, 7.76	0.00, 7.26	0.00, 6.26
6	2.21,12.75	1.90,12.25	1.61,11.75	1.11,11.25	0.61,10.75	0.11,10.25	0.00, 9.75	0.00, 9.25	0.00, 8.75	0.00, 7.75
7	2.58,13.81	2.27,13.31	1.97,12.81	1.69,12.31	1.29,11.81	0.79,11.31	0.29,10.81	0.00,10.31	0.00, 9.81	0.00, 8.81
8	2.94,15.29	2.63,14.79	2.33,14.29	2.05,13.79	1.78,13.29	1.48,12.79	0.98,12.29	0.48,11.79	0.00,11.29	0.00,10.29
9	4.36,16.77	3.86,16.27	3.36,15.77	2.91,15.27	2.46,14.77	1.96,14.27	1.62,13.77	1.20,13.27	0.70,12.77	0.00,11.77
10	4.75,17.82	4.25,17.32	3.75,16.82	3.30,16.32	2.92,15.82	2.57,15.32	2.25,14.82	1.82,14.32	1.43,13.82	0.43,12.82
11	5.14,19.29	4.64,18.79	4.14,18.29	3.69,17.79	3.30,17.29	2.95,16.79	2.63,16.29	2.33,15.79	2.04,15.29	1.17,14.29
12	6.32,20.34	5.82,19.84	5.32,19.34	4.82,18.84	4.32,18.34	3.85,17.84	3.44,17.34	3.06,16.84	2.69,16.34	1.88,15.34
13	6.72,21.80	6.22,21.30	5.72,20.80	5.22,20.30	4.72,19.80	4.25,19.30	3.84,18.80	3.46,18.30	3.11,17.80	2.47,16.80
14	7.84,22.94	7.34,22.44	6.84,21.94	6.34,21.44	5.84,20.94	5.34,20.44	4.84,19.94	4.37,19.44	3.94,18.94	3.10,17.94
15	8.25,24.31	7.75,23.81	7.25,23.31	6.75,22.81	6.25,22.31	5.75,21.81	5.25,21.31	4.78,20.81	4.35,20.31	3.58,19.31
16	9.34,25.40	8.84,24.90	8.34,24.40	7.84,23.90	7.34,23.40	6.84,22.90	6.34,22.40	5.84,21.90	5.34,21.40	4.43,20.40
17	9.76,26.81	9.26,26.31	8.76,25.81	8.26,25.31	7.76,24.81	7.26,24.31	6.76,23.81	6.26,23.31	5.76,22.81	4.84,21.81
18	10.84,27.84	10.34,27.34	9.84,26.84	9.34,26.34	8.84,25.84	8.34,25.34	7.84,24.84	7.34,24.34	6.84,23.84	5.84,22.84
19	11.26,29.31	10.76,28.81	10.26,28.31	9.76,27.81	9.26,27.31	8.76,26.81	8.26,26.31	7.76,25.81	7.26,25.31	6.26,24.31
20	12.33,30.33	11.83,29.83	11.33,29.33	10.83,28.83	10.33,28.33	9.83,27.83	9.33,27.33	8.83,26.83	8.33,26.33	7.33,25.33

TABLE VIII. 99% C.L. intervals for the Poisson signal mean μ, for total events observed n_0, for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 4.74	0.00, 4.24	0.00, 3.80	0.00, 3.50	0.00, 3.26	0.00, 3.26	0.00, 3.05	0.00, 3.05	0.00, 2.98	*0.00, 2.94*
1	0.01, 6.91	0.00, 6.41	0.00, 5.91	0.00, 5.41	0.00, 4.91	0.00, 4.48	0.00, 4.14	0.00, 4.09	0.00, 3.89	0.00, 3.59
2	0.15, 8.71	0.00, 8.21	0.00, 7.71	0.00, 7.21	0.00, 6.71	0.00, 6.24	0.00, 5.82	0.00, 5.42	0.00, 5.06	0.00, 4.37
3	0.44,10.47	0.00, 9.97	0.00, 9.47	0.00, 8.97	0.00, 8.47	0.00, 7.97	0.00, 7.47	0.00, 6.97	0.00, 6.47	0.00, 5.57
4	0.82,12.23	0.32,11.73	0.00,11.23	0.00,10.73	0.00,10.23	0.00, 9.73	0.00, 9.23	0.00, 8.73	0.00, 8.23	0.00, 7.30
5	1.28,13.75	0.78,13.25	0.28,12.75	0.00,12.25	0.00,11.75	0.00,11.25	0.00,10.75	0.00,10.25	0.00, 9.75	0.00, 8.75
6	1.79,15.27	1.29,14.77	0.79,14.27	0.29,13.77	0.00,13.27	0.00,12.77	0.00,12.27	0.00,11.77	0.00,11.27	0.00,10.27
7	2.33,16.77	1.83,16.27	1.33,15.77	0.83,15.27	0.33,14.77	0.00,14.27	0.00,13.77	0.00,13.27	0.00,12.77	0.00,11.77
8	2.91,18.27	2.41,17.77	1.91,17.27	1.41,16.77	0.91,16.27	0.41,15.77	0.00,15.27	0.00,14.77	0.00,14.27	0.00,13.27
9	3.31,19.46	3.00,18.96	2.51,18.46	2.01,17.96	1.51,17.46	1.01,16.96	0.51,16.46	0.01,15.96	0.00,15.46	0.00,14.46
10	3.68,20.83	3.37,20.33	3.07,19.83	2.63,19.33	2.13,18.83	1.63,18.33	1.13,17.83	0.63,17.33	0.13,16.83	0.00,15.83
11	4.05,22.31	3.73,21.81	3.43,21.31	3.14,20.81	2.77,20.31	2.27,19.81	1.77,19.31	1.27,18.81	0.77,18.31	0.00,17.31
12	4.41,23.80	4.10,23.30	3.80,22.80	3.50,22.30	3.22,21.80	2.93,21.30	2.43,20.80	1.93,20.30	1.43,19.80	0.43,18.80
13	5.83,24.92	5.33,24.42	4.83,23.92	4.33,23.42	3.83,22.92	3.33,22.42	3.02,21.92	2.60,21.42	2.10,20.92	1.10,19.92
14	6.31,26.33	5.81,25.83	5.31,25.33	4.86,24.83	4.46,24.33	4.10,23.83	3.67,23.33	3.17,22.83	2.78,22.33	1.78,21.33
15	6.70,27.81	6.20,27.31	5.70,26.81	5.24,26.31	4.84,25.81	4.48,25.31	4.14,24.81	3.82,24.31	3.42,23.81	2.48,22.81
16	7.76,28.85	7.26,28.35	6.76,27.85	6.26,27.35	5.76,26.85	5.26,26.35	4.76,25.85	4.26,25.35	3.89,24.85	3.15,23.85
17	8.32,30.33	7.82,29.83	7.32,29.33	6.82,28.83	6.32,28.33	5.85,27.83	5.42,27.33	5.03,26.83	4.67,26.33	3.73,25.33
18	8.71,31.81	8.21,31.31	7.71,30.81	7.21,30.31	6.71,29.81	6.24,29.31	5.82,28.81	5.42,28.31	5.06,27.81	4.37,26.81
19	9.88,32.85	9.38,32.35	8.88,31.85	8.38,31.35	7.88,30.85	7.38,30.35	6.88,29.85	6.40,29.35	5.97,28.85	5.01,27.85
20	10.28,34.32	9.78,33.82	9.28,33.32	8.78,32.82	8.28,32.32	7.78,31.82	7.28,31.32	6.81,30.82	6.37,30.32	5.57,29.32