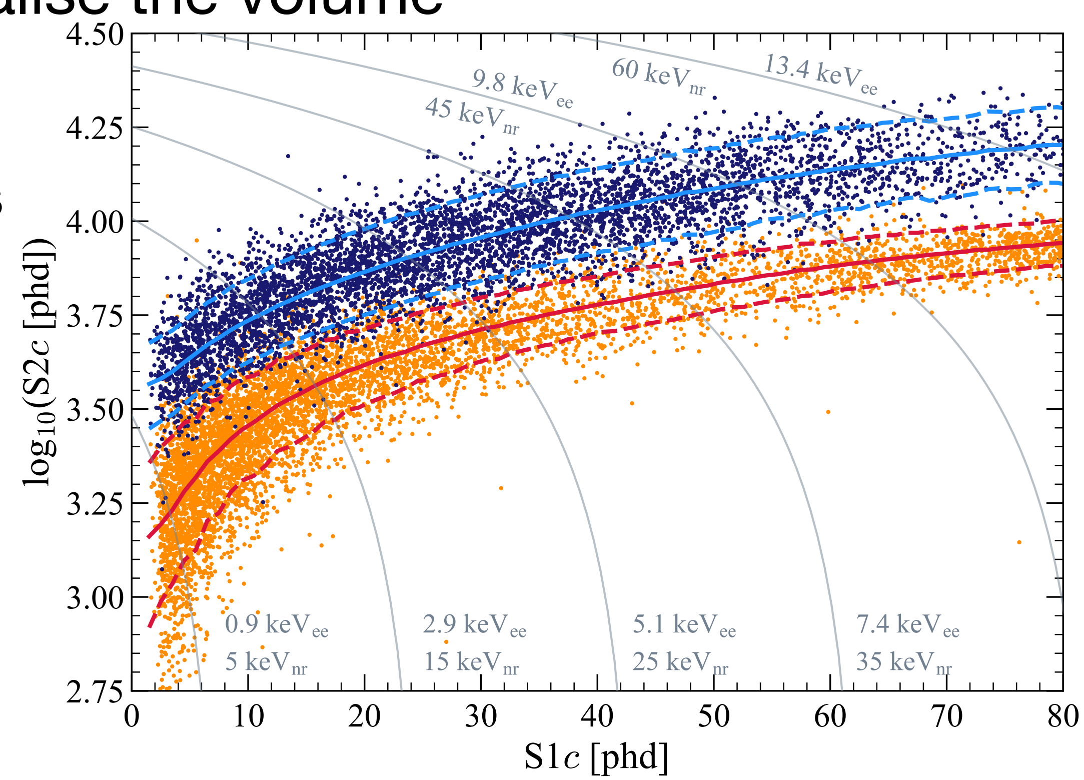
Lesson 4

Exercises

Alexandre Lindote, 22nd of March 2024

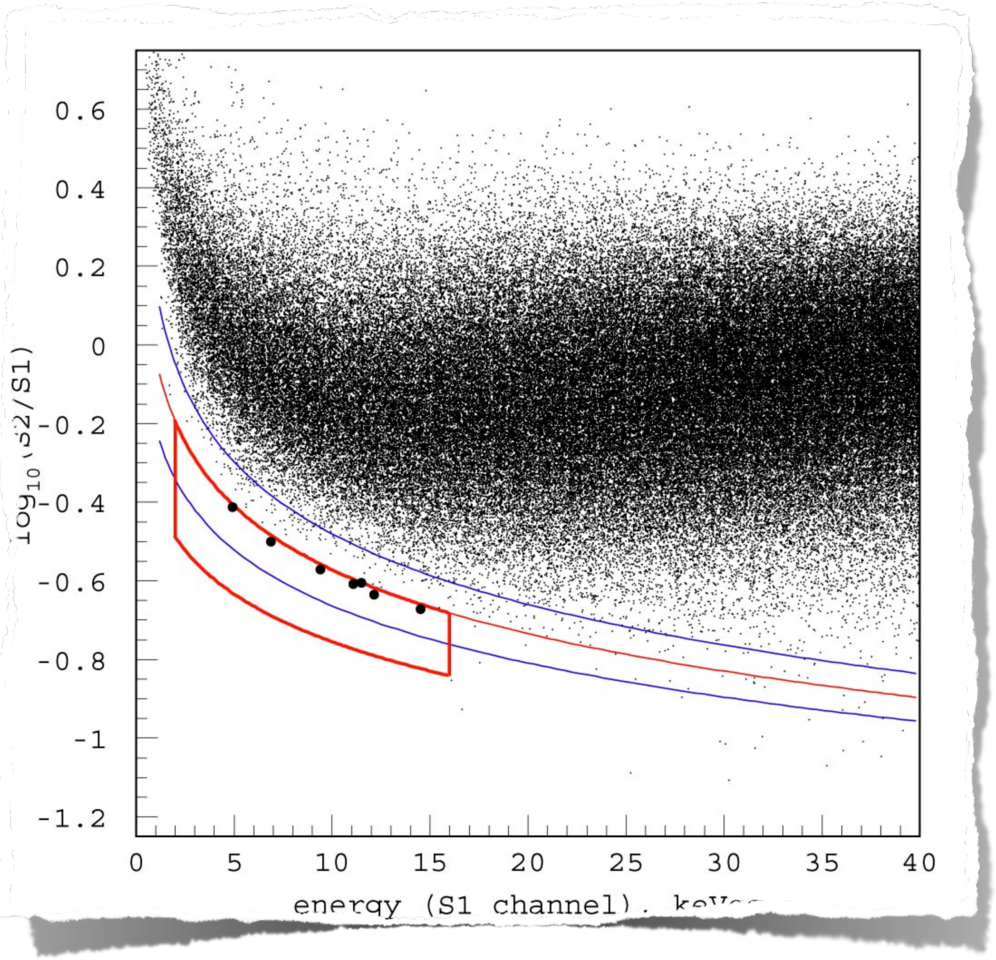
The Experiment

- Consider a WIMP search experiment based on a 2-phase xenon TPC
  - Assume that the fiducial mass is 5600 kg
  - And that the experiment runs for 3 years with an average 90% duty cycle
- As with any similar detector, the experiment measures the S1 (scintillation) and S2 (ionisation) signals for each interaction
  - 3D position reconstruction => ability to fiducialise the volume
  - NR vs ER discrimination (based on S2/S1)
    - Assume a **50% acceptance** for nuclear recoils



The Experiment

- This experiment is searching for NRs in the **5-50 keV** nuclear recoil range
- Let's treat this as a simplified counting experiment
  - After a thorough analysis of all the background sources (ER and NR), we expect **1.5** events in our search region (<u>before unblinding</u> the science data)
  - After unblinding, we observe **4** candidate events!



- For simplicity, let's assume
  - The Earth is stationary: $\boldsymbol{v_E} = (0, 0, 0)$
  - There is no upper limit on the velocity of the DM particles:
  - The form factor is 1: $F^2(q) = 1$

Exercise 1

- Find the 90% confidence level interval on the mean number of signal counts (μ) from Feldman-Cousins* statistics
  - Would you claim a WIMP discovery with this experiment?
  - Justify

* G.J. Feldman, R.D. Cousins, “A Unified Approach to the Classical Statistical Analysis of Small Signals”, PRD 57 (1998) 3873, arXiv:physics/9711021

Exercise 1

Feldman-Cousins statistics

TABLE IV. 90% C.L. intervals for the Poisson signal mean μ, for total events observed n_0 , for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 2.44	0.00, 1.94	0.00, 1.61	0.00, 1.33	0.00, 1.26	0.00, 1.18	0.00, 1.08	0.00, 1.06	0.00, 1.01	*0.00, 0.98*
1	0.11, 4.36	0.00, 3.86	0.00, 3.36	0.00, 2.91	0.00, 2.53	0.00, 2.19	0.00, 1.88	0.00, 1.59	0.00, 1.39	0.00, 1.22
2	0.53, 5.91	0.03, 5.41	0.00, 4.91	0.00, 4.41	0.00, 3.91	0.00, 3.45	0.00, 3.04	0.00, 2.67	0.00, 2.33	0.00, 1.73
3	1.10, 7.42	0.60, 6.92	0.10, 6.42	0.00, 5.92	0.00, 5.42	0.00, 4.92	0.00, 4.42	0.00, 3.95	0.00, 3.53	0.00, 2.78
4	1.47, 8.60	1.17, 8.10	0.74, 7.60	0.24, 7.10	0.00, 6.60	0.00, 6.10	0.00, 5.60	0.00, 5.10	0.00, 4.60	0.00, 3.60
5	1.84, 9.99	1.53, 9.49	1.25, 8.99	0.93, 8.49	0.43, 7.99	0.00, 7.49	0.00, 6.99	0.00, 6.49	0.00, 5.99	0.00, 4.99
6	2.21,11.47	1.90,10.97	1.61,10.47	1.33, 9.97	1.08, 9.47	0.65, 8.97	0.15, 8.47	0.00, 7.97	0.00, 7.47	0.00, 6.47
7	3.56,12.53	3.06,12.03	2.56,11.53	2.09,11.03	1.59,10.53	1.18,10.03	0.89, 9.53	0.39, 9.03	0.00, 8.53	0.00, 7.53
8	3.96,13.99	3.46,13.49	2.96,12.99	2.51,12.49	2.14,11.99	1.81,11.49	1.51,10.99	1.06,10.49	0.66, 9.99	0.00, 8.99
9	4.36,15.30	3.86,14.80	3.36,14.30	2.91,13.80	2.53,13.30	2.19,12.80	1.88,12.30	1.59,11.80	1.33,11.30	0.43,10.30
10	5.50,16.50	5.00,16.00	4.50,15.50	4.00,15.00	3.50,14.50	3.04,14.00	2.63,13.50	2.27,13.00	1.94,12.50	1.19,11.50
11	5.91,17.81	5.41,17.31	4.91,16.81	4.41,16.31	3.91,15.81	3.45,15.31	3.04,14.81	2.67,14.31	2.33,13.81	1.73,12.81
12	7.01,19.00	6.51,18.50	6.01,18.00	5.51,17.50	5.01,17.00	4.51,16.50	4.01,16.00	3.54,15.50	3.12,15.00	2.38,14.00
13	7.42,20.05	6.92,19.55	6.42,19.05	5.92,18.55	5.42,18.05	4.92,17.55	4.42,17.05	3.95,16.55	3.53,16.05	2.78,15.05
14	8.50,21.50	8.00,21.00	7.50,20.50	7.00,20.00	6.50,19.50	6.00,19.00	5.50,18.50	5.00,18.00	4.50,17.50	3.59,16.50
15	9.48,22.52	8.98,22.02	8.48,21.52	7.98,21.02	7.48,20.52	6.98,20.02	6.48,19.52	5.98,19.02	5.48,18.52	4.48,17.52
16	9.99,23.99	9.49,23.49	8.99,22.99	8.49,22.49	7.99,21.99	7.49,21.49	6.99,20.99	6.49,20.49	5.99,19.99	4.99,18.99
17	11.04,25.02	10.54,24.52	10.04,24.02	9.54,23.52	9.04,23.02	8.54,22.52	8.04,22.02	7.54,21.52	7.04,21.02	6.04,20.02
18	11.47,26.16	10.97,25.66	10.47,25.16	9.97,24.66	9.47,24.16	8.97,23.66	8.47,23.16	7.97,22.66	7.47,22.16	6.47,21.16
19	12.51,27.51	12.01,27.01	11.51,26.51	11.01,26.01	10.51,25.51	10.01,25.01	9.51,24.51	9.01,24.01	8.51,23.51	7.51,22.51
20	13.55,28.52	13.05,28.02	12.55,27.52	12.05,27.02	11.55,26.52	11.05,26.02	10.55,25.52	10.05,25.02	9.55,24.52	8.55,23.52

Exercise 1

Feldman-Cousins statistics

TABLE IV. 90% C.L. intervals for the Poisson signal mean μ, for total events observed n_0, for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 2.44	0.00, 1.94	0.00, 1.61	0.00, 1.33	0.00, 1.26	0.00, 1.18	0.00, 1.08	0.00, 1.06	0.00, 1.01	*0.00, 0.98*
1	0.11, 4.36	0.00, 3.86	0.00, 3.36	0.00, 2.91	0.00, 2.53	0.00, 2.19	0.00, 1.88	0.00, 1.59	0.00, 1.39	0.00, 1.22
2	0.53, 5.91	0.03, 5.41	0.00, 4.91	0.00, 4.41	0.00, 3.91	0.00, 3.45	0.00, 3.04	0.00, 2.67	0.00, 2.33	0.00, 1.73
3	1.10, 7.42	0.60, 6.92	0.10, 6.42	0.00, [illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0.00, 3.53	0.00, 2.78
4	1.47, 8.60	1.17, 8.10	0.74, 7.60	0.24, 7.10	0.00, 6.60	0.00, 6.10	0.00, 5.60	0.00, 5.10	0.00, 4.60	0.00, 3.60
5	1.84, 9.99	1.53, 9.49	1.25, 8.99	0.93, 8.49	0.43, 7.99	0.00, 7.49	0.00, 6.99	0.00, 6.49	0.00, 5.99	0.00, 4.99
6	2.21,11.47	1.90,10.97	1.61,10.47	1.33, 9.97	1.08, 9.47	0.65, 8.97	0.15, 8.47	0.00, 7.97	0.00, 7.47	0.00, 6.47
7	3.56,12.53	3.06,12.03	2.56,11.53	2.09,11.03	1.59,10.53	1.18,10.03	0.89, 9.53	0.39, 9.03	0.00, 8.53	0.00, 7.53
8	3.96,13.99	3.46,13.49	2.96,12.99	2.51,12.49	2.14,11.99	1.81,11.49	1.51,10.99	1.06,10.49	0.66, 9.99	0.00, 8.99
9	4.36,15.30	3.86,14.80	3.36,14.30	2.91,13.80	2.53,13.30	2.19,12.80	1.88,12.30	1.59,11.80	1.33,11.30	0.43,10.30
10	5.50,16.50	5.00,16.00	4.50,15.50	4.00,15.00	3.50,14.50	3.04,14.00	2.63,13.50	2.27,13.00	1.94,12.50	1.19,11.50
11	5.91,17.81	5.41,17.31	4.91,16.81	4.41,16.31	3.91,15.81	3.45,15.31	3.04,14.81	2.67,14.31	2.33,13.81	1.73,12.81
12	7.01,19.00	6.51,18.50	6.01,18.00	5.51,17.50	5.01,17.00	4.51,16.50	4.01,16.00	3.54,15.50	3.12,15.00	2.38,14.00
13	7.42,20.05	6.92,19.55	6.42,19.05	5.92,18.55	5.42,18.05	4.92,17.55	4.42,17.05	3.95,16.55	3.53,16.05	2.78,15.05
14	8.50,21.50	8.00,21.00	7.50,20.50	7.00,20.00	6.50,19.50	6.00,19.00	5.50,18.50	5.00,18.00	4.50,17.50	3.59,16.50
15	9.48,22.52	8.98,22.02	8.48,21.52	7.98,21.02	7.48,20.52	6.98,20.02	6.48,19.52	5.98,19.02	5.48,18.52	4.48,17.52
16	9.99,23.99	9.49,23.49	8.99,22.99	8.49,22.49	7.99,21.99	7.49,21.49	6.99,20.99	6.49,20.49	5.99,19.99	4.99,18.99
17	11.04,25.02	10.54,24.52	10.04,24.02	9.54,23.52	9.04,23.02	8.54,22.52	8.04,22.02	7.54,21.52	7.04,21.02	6.04,20.02
18	11.47,26.16	10.97,25.66	10.47,25.16	9.97,24.66	9.47,24.16	8.97,23.66	8.47,23.16	7.97,22.66	7.47,22.16	6.47,21.16
19	12.51,27.51	12.01,27.01	11.51,26.51	11.01,26.01	10.51,25.51	10.01,25.01	9.51,24.51	9.01,24.01	8.51,23.51	7.51,22.51
20	13.55,28.52	13.05,28.02	12.55,27.52	12.05,27.02	11.55,26.52	11.05,26.02	10.55,25.52	10.05,25.02	9.55,24.52	8.55,23.52

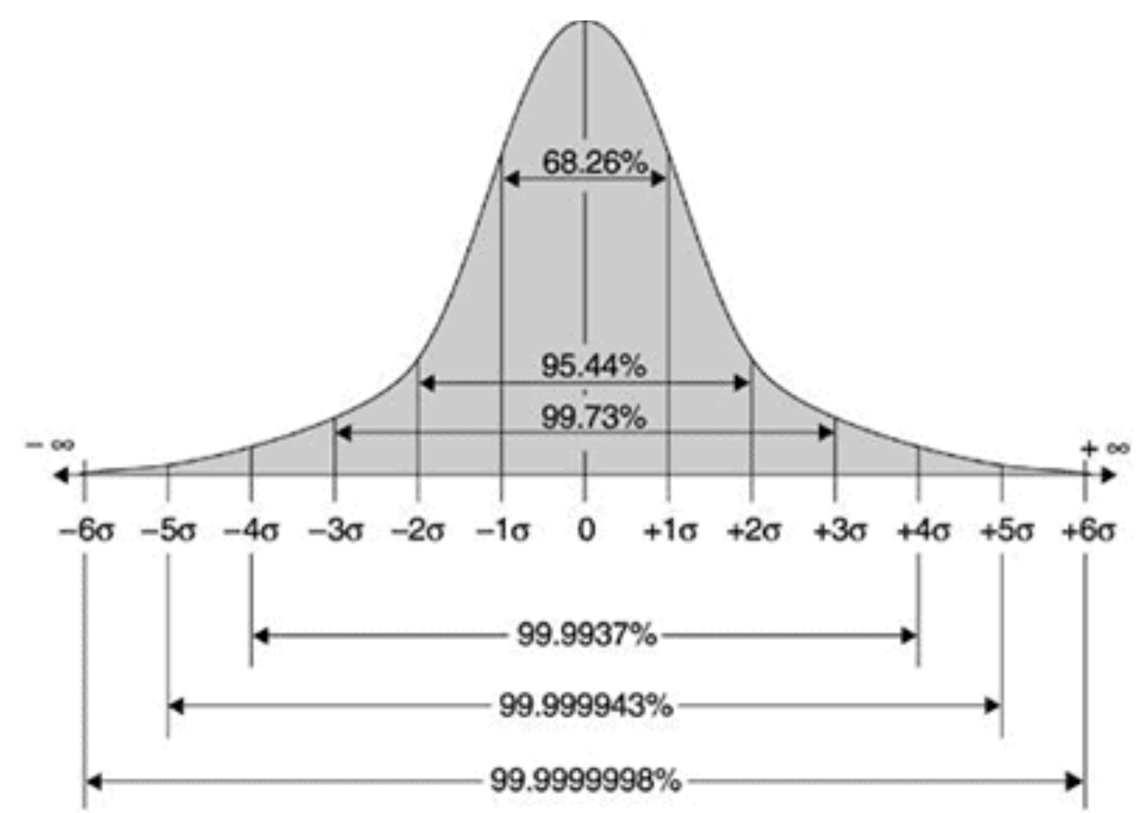
At 90% CL there seems to be a signal!

But is it enough to claim a discovery?…

Exercise 1

Feldman-Cousins statistics

- In science, a 90% CL is not enough to declare a “discovery”
- Definitions vary, but in high-energy physics the accepted levels are:
    - 3σ to be considered an “observation”
    - 5σ to be considered a “discovery”
- Both are well above a 90% CL:
    - 3σ is a 99.73% CL
    - 5σ is a 99.999943% CL



Exercise 1

Feldman-Cousins statistics

- If we look in the 95% CL table, these results are already compatible with no signal !

TABLE VI. 95% C.L. intervals for the Poisson signal mean μ, for total events observed n_0 , for known mean background b ranging from 0 to 5.

$n_0 \backslash b$	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0
0	0.00, 3.09	0.00, 2.63	0.00, 2.33	0.00, 2.05	0.00, 1.78	0.00, 1.78	0.00, 1.63	0.00, 1.63	0.00, 1.57	*0.00, 1.54*
1	0.05, 5.14	0.00, 4.64	0.00, 4.14	0.00, 3.69	0.00, 3.30	0.00, 2.95	0.00, 2.63	0.00, 2.33	0.00, 2.08	0.00, 1.88
2	0.36, 6.72	0.00, 6.22	0.00, 5.72	0.00, 5.22	0.00, 4.72	0.00, 4.25	0.00, 3.84	0.00, 3.46	0.00, 3.11	0.00, 2.49
3	0.82, 8.25	0.32, 7.75	0.00, 7.25	0.00, 6.75	0.00, 6.25	0.00, 5.75	0.00, 5.25	0.00, 4.78	0.00, 4.35	0.00, 3.58
4	1.37, 9.76	0.87, 9.26	0.37, 8.76	0.00, 8.26	0.00, 7.76	0.00, 7.26	0.00, 6.76	0.00, 6.26	0.00, 5.76	0.00, 4.84
5	1.84,11.26	1.47,10.76	0.97,10.26	0.47, 9.76	0.00, 9.26	0.00, 8.76	0.00, 8.26	0.00, 7.76	0.00, 7.26	0.00, 6.26
6	2.21,12.75	1.90,12.25	1.61,11.75	1.11,11.25	0.61,10.75	0.11,10.25	0.00, 9.75	0.00, 9.25	0.00, 8.75	0.00, 7.75
7	2.58,13.81	2.27,13.31	1.97,12.81	1.69,12.31	1.29,11.81	0.79,11.31	0.29,10.81	0.00,10.31	0.00, 9.81	0.00, 8.81
8	2.94,15.29	2.63,14.79	2.33,14.29	2.05,13.79	1.78,13.29	1.48,12.79	0.98,12.29	0.48,11.79	0.00,11.29	0.00,10.29
9	4.36,16.77	3.86,16.27	3.36,15.77	2.91,15.27	2.46,14.77	1.96,14.27	1.62,13.77	1.20,13.27	0.70,12.77	0.00,11.77
10	4.75,17.82	4.25,17.32	3.75,16.82	3.30,16.32	2.92,15.82	2.57,15.32	2.25,14.82	1.82,14.32	1.43,13.82	0.43,12.82
11	5.14,19.29	4.64,18.79	4.14,18.29	3.69,17.79	3.30,17.29	2.95,16.79	2.63,16.29	2.33,15.79	2.04,15.29	1.17,14.29
12	6.32,20.34	5.82,19.84	5.32,19.34	4.82,18.84	4.32,18.34	3.85,17.84	3.44,17.34	3.06,16.84	2.69,16.34	1.88,15.34
13	6.72,21.80	6.22,21.30	5.72,20.80	5.22,20.30	4.72,19.80	4.25,19.30	3.84,18.80	3.46,18.30	3.11,17.80	2.47,16.80
14	7.84,22.94	7.34,22.44	6.84,21.94	6.34,21.44	5.84,20.94	5.34,20.44	4.84,19.94	4.37,19.44	3.94,18.94	3.10,17.94
15	8.25,24.31	7.75,23.81	7.25,23.31	6.75,22.81	6.25,22.31	5.75,21.81	5.25,21.31	4.78,20.81	4.35,20.31	3.58,19.31
16	9.34,25.40	8.84,24.90	8.34,24.40	7.84,23.90	7.34,23.40	6.84,22.90	6.34,22.40	5.84,21.90	5.34,21.40	4.43,20.40
17	9.76,26.81	9.26,26.31	8.76,25.81	8.26,25.31	7.76,24.81	7.26,24.31	6.76,23.81	6.26,23.31	5.76,22.81	4.84,21.81
18	10.84,27.84	10.34,27.34	9.84,26.84	9.34,26.34	8.84,25.84	8.34,25.34	7.84,24.84	7.34,24.34	6.84,23.84	5.84,22.84
19	11.26,29.31	10.76,28.81	10.26,28.31	9.76,27.81	9.26,27.31	8.76,26.81	8.26,26.31	7.76,25.81	7.26,25.31	6.26,24.31
20	12.33,30.33	11.83,29.83	11.33,29.33	10.83,28.83	10.33,28.33	9.83,27.83	9.33,27.33	8.83,26.83	8.33,26.33	7.33,25.33

WIMP-nucleus cross section limit

Exercise 2: Estimate the experiment sensitivity for a 50 GeV WIMP

- Start from the differential nuclear recoil spectrum we expect from WIMPs (slide 18 in Lesson 2)

$$\frac{dR}{dE_R} = \frac{\rho_0 \sigma_A}{2 m_\chi \mu_A^2} F^2(q) \int_{v_{\min}}^{v_{\max}} \frac{f(\vec{v})}{v} d^3 v$$

- dR is the NR differential rate (ev/kg/day/keV)
- dE_R is our energy range

WIMP-nucleus cross section limit

Exercise 2: Estimate the experiment sensitivity for a 50 GeV WIMP

- Start from the differential nuclear recoil spectrum we expect from WIMPs (slide 18 in Lesson 2)

$$\frac{dR}{dE_R} = \frac{\rho_0 \sigma_A}{2 m_\chi \mu_A^2} F^2(q) \int_{v_{\min}}^{v_{\max}} \frac{f(\vec{v})}{v} d^3 v$$

- dR is the NR differential rate (ev/kg/day/keV)
- dE_R is our energy range

Let's assume some simplifications:

- The Earth is stationary
- There is no upper limit on the velocity of the DM particles:
- The form factor is 1: $F^2(q) = 1$

WIMP-nucleus cross section limit

Exercise 2: Estimate the experiment sensitivity for a 50 GeV WIMP

- The differential nuclear recoil spectrum can be approximated to (see Eq. 3.10 in the Lewin&Smith paper, see bibliography)

$$\frac{dR}{dE_R} = \frac{\rho_0 \sigma_A}{2 m_\chi \mu_A^2} F^2(q) \int_{v_{\min}}^{v_{\max}} \frac{f(\vec{v})}{v} d^3 v \sim \frac{R_0}{E_0 r} e^{-\frac{E_R}{E_0 r}}$$

- dR is the NR differential rate (ev/kg/day/keV)
- dE_R is our energy range
- R_0 is a reference integral rate (normalisation)
- E_0 is the most probable WIMP kinetic energy
- r is the kinematic factor

$$R_0 = \frac{2 \rho_0 \sigma_A v_0}{m_\chi m_A \sqrt{\pi}}$$

$$E_i = \frac{1}{2} m_\chi \mathrm{v}^2$$

$$r = \frac{4 m_\chi m_N}{(m_\chi + m_N)^2}$$

1 GeV/c^2 = 1.78266 x 10^{-27} kg

WIMP-nucleon cross section limit

Exercise 3 (use the WIMP-nucleus CS obtained in exercise 2)

- From slide 22 of lesson 2 (or from L&S paper)

$$\sigma_A^{SI}(q \to 0) = \frac{4\mu_A^2}{\pi}[Zf_p + (A-Z)f_n]^2 \approx \frac{\mu_A^2}{\mu_p^2}\sigma_p A^2$$

- Reduced masses: $\mu = \dfrac{m_\chi m_N}{m_\chi + m_N}$

- m_{Xe} = 122.3 GeV/c^2
- m_p = 0.938 GeV/c^2
- A = 131.2

Sensitivity curve
Homework

- Generalize the calculations we have just done. Create a plot of the experiment sensitivity (at 90%, 95% or 99% CL) to the SI WIMP-nucleon interaction cross section as a function of the WIMP mass (1 GeV to 1 TeV)

- Feel free to use your favourite tool! Use Python, GNUPlot, Excel, Google Sheets, etc. You can even do it manually!

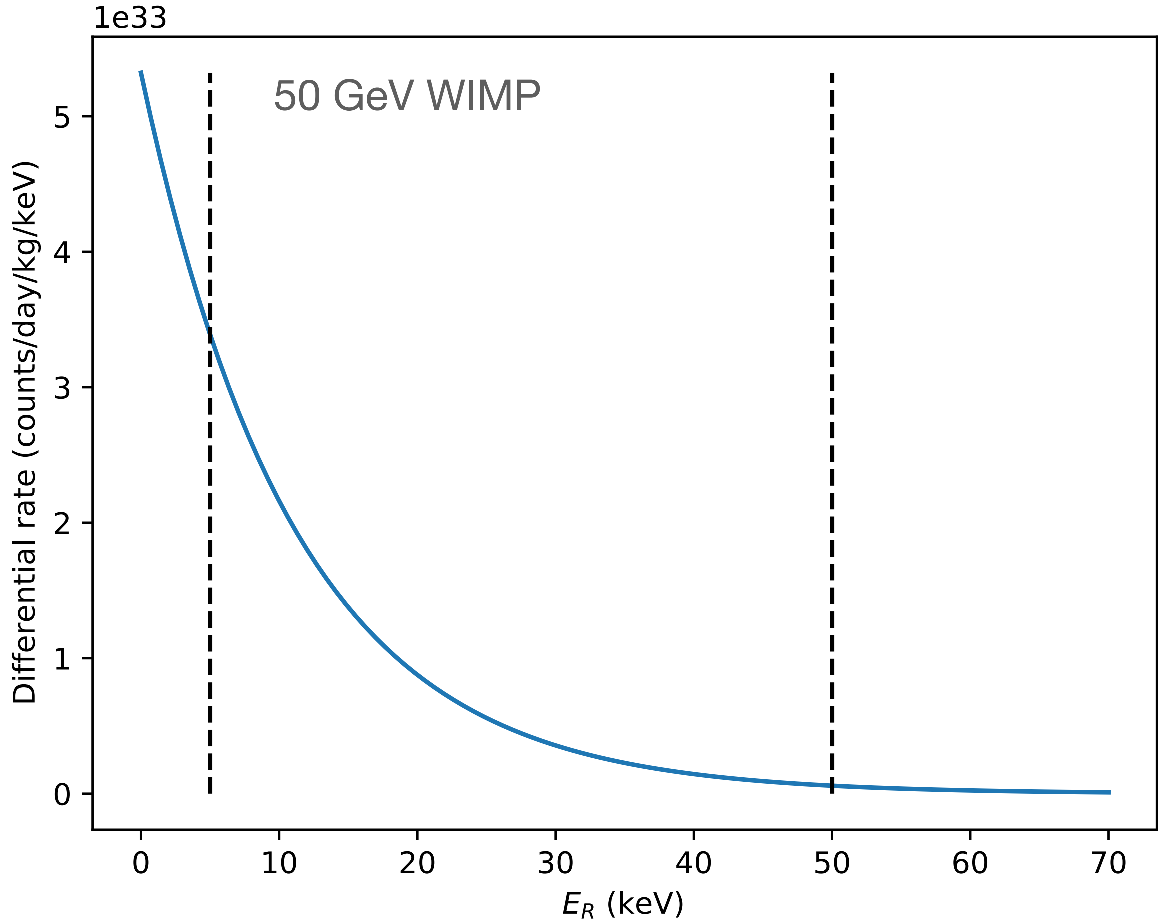
Exercise 4

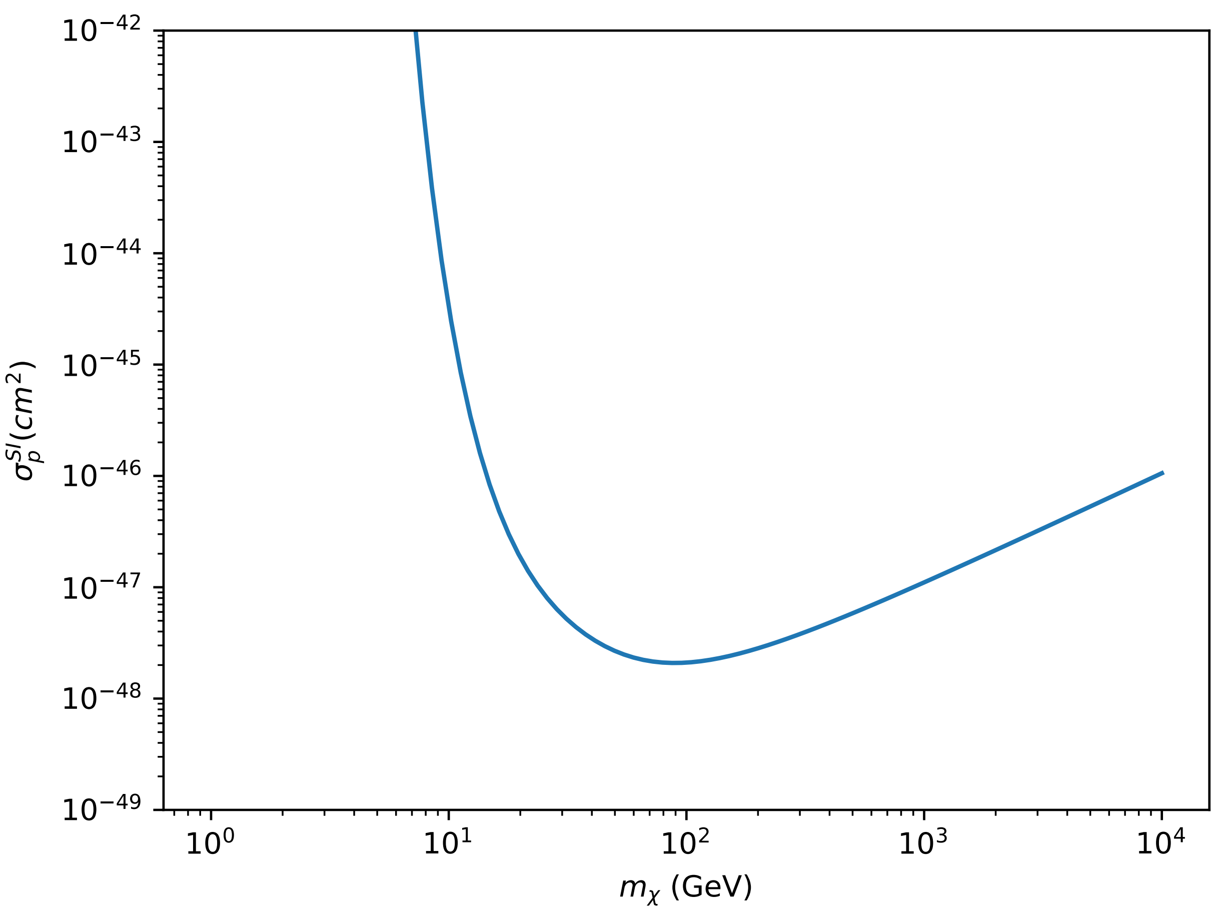
Minimum WIMP mass

- What is the minimum WIMP mass our detector is sensitive to?
  - Assume a WIMP escape velocity of 544 km/s
  - Assume the velocity of the Earth is 220 km/s

Effect of the various simplifications

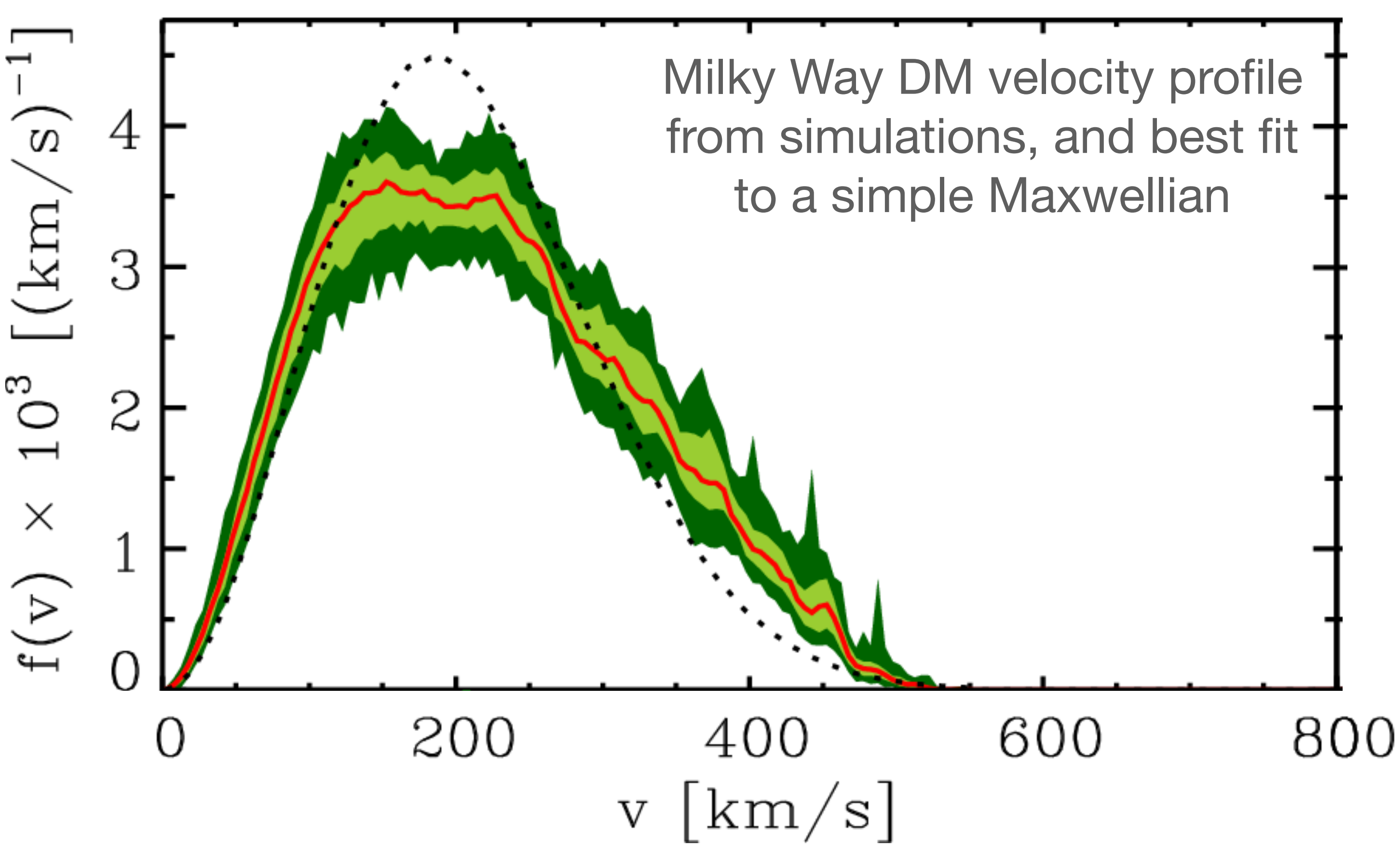
- These are the differential rate and 90% CL limit on the cross section plots with the assumptions we used

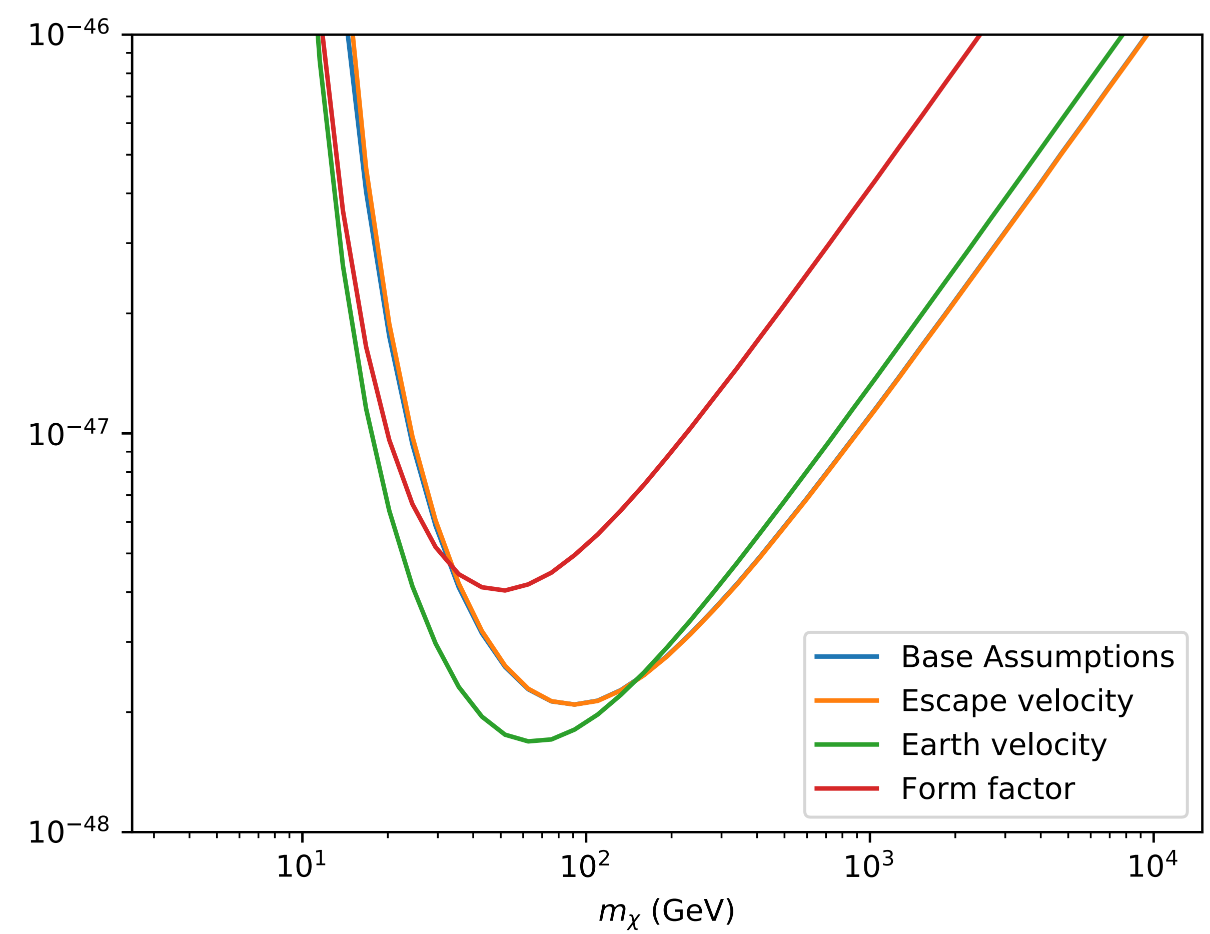




Effect of the various simplifications

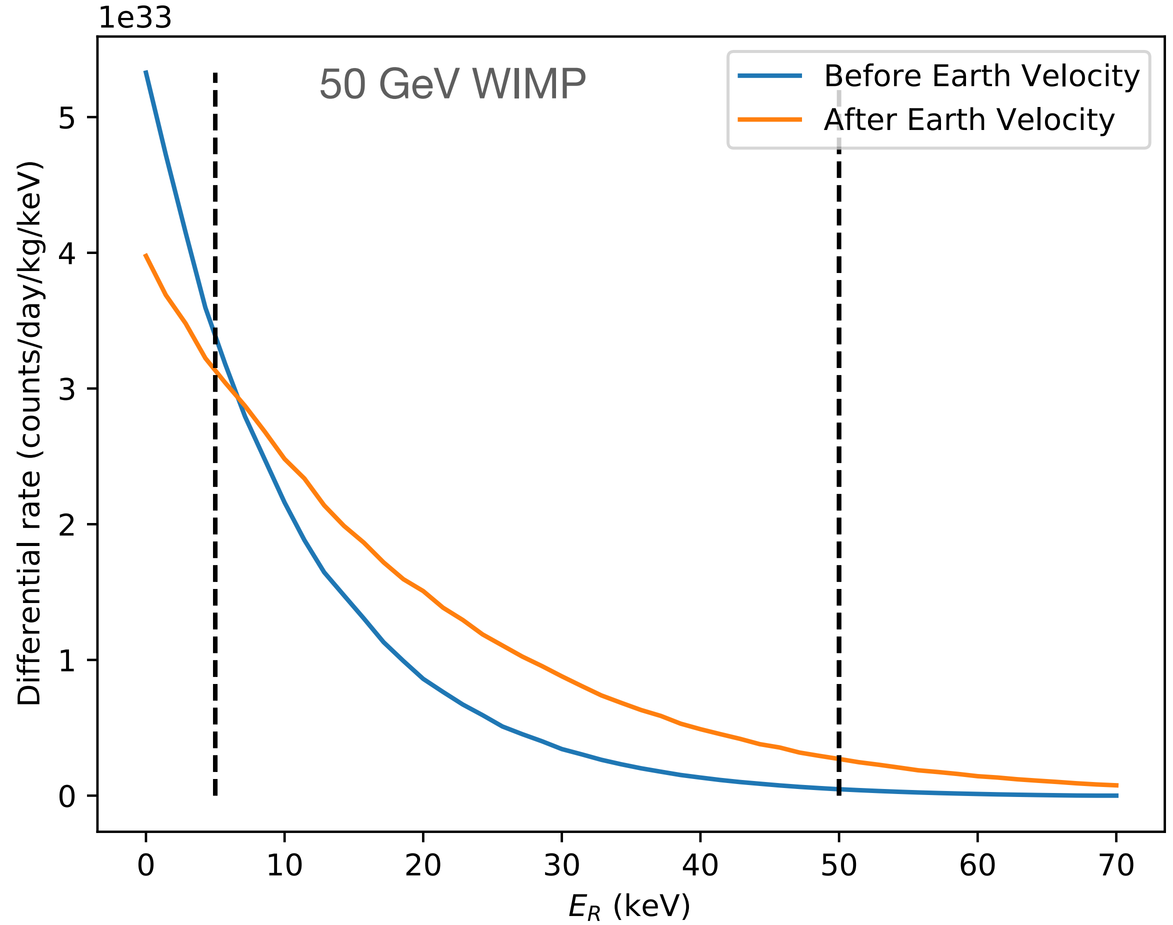
- Including a finite escape velocity has basically no effect

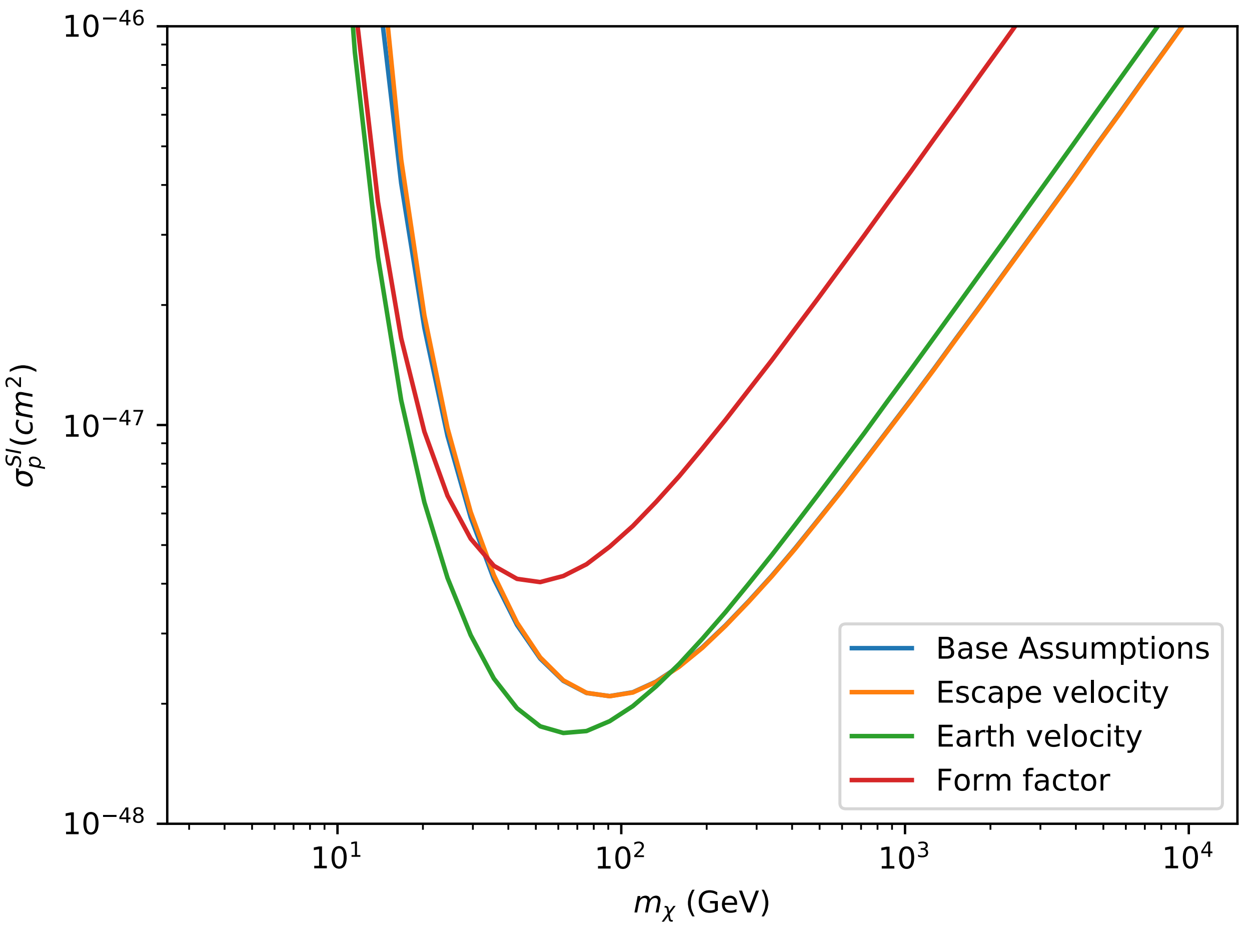




Effect of the various simplifications

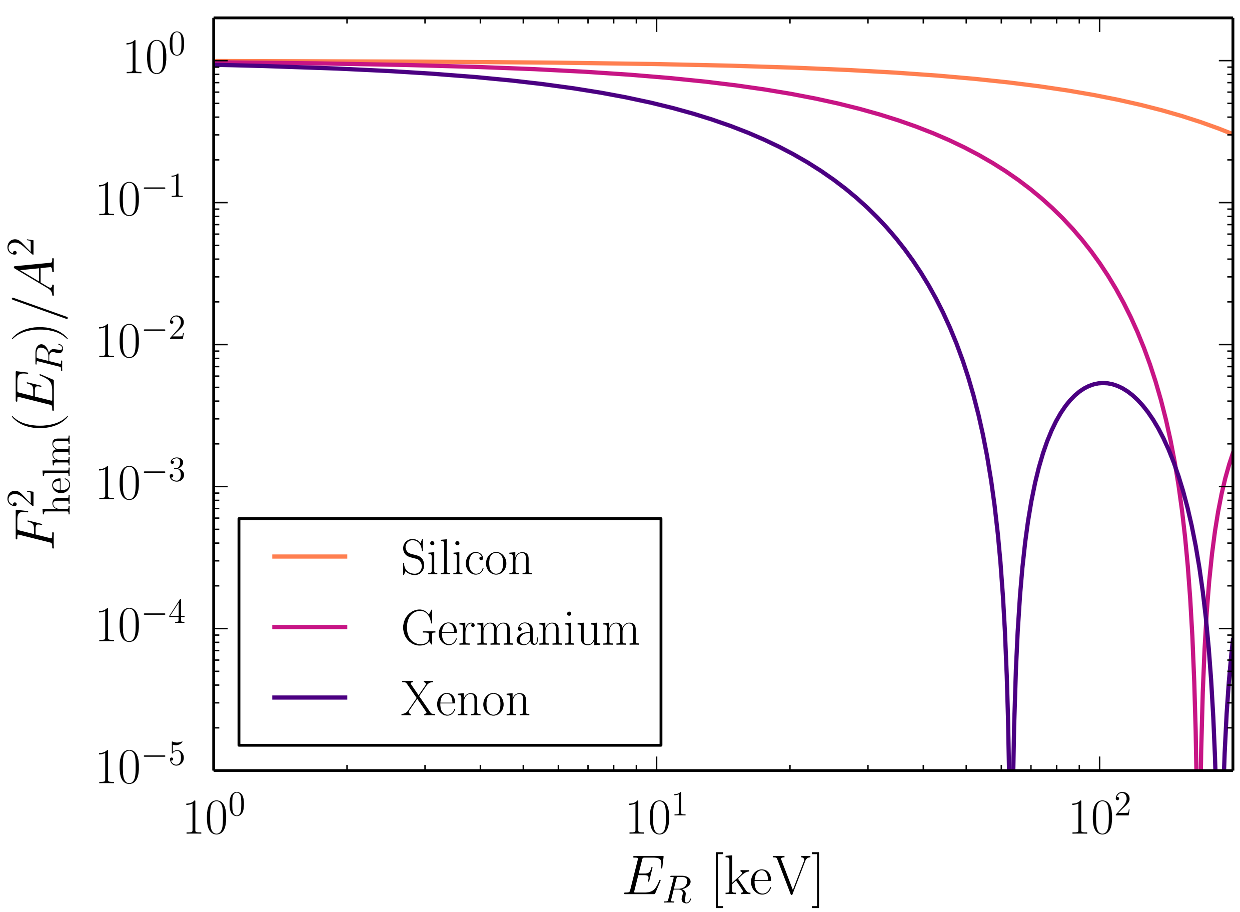
- Including a finite escape velocity has basically no effect
- Adding the Earth velocity clearly increases the recoil spectrum in the chosen energy range (WIMP particles have more energy), making the detection more favourable for low masses

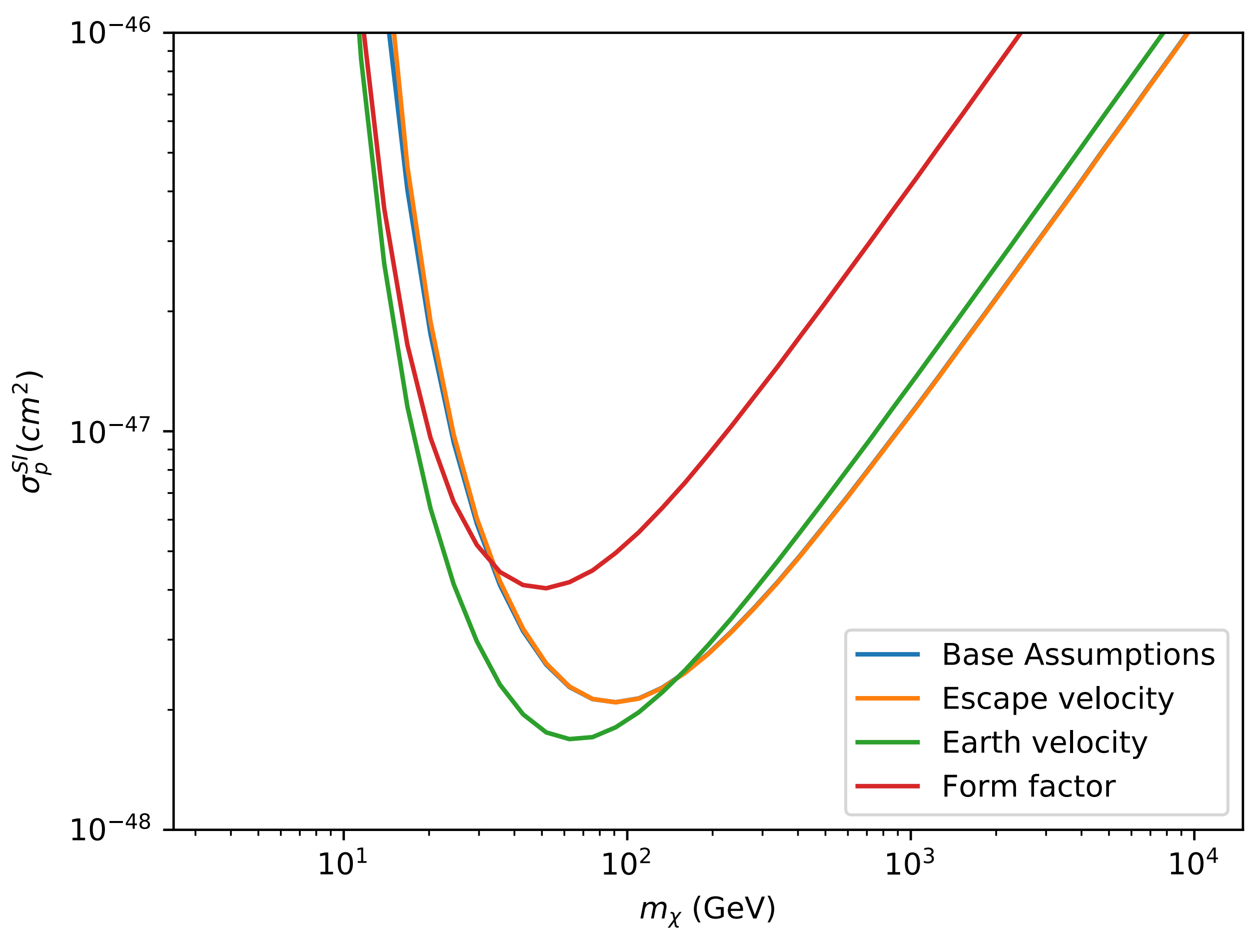




Effect of the various simplifications

- Including a finite escape velocity has basically no effect
- Adding the Earth velocity clearly increases the recoil spectrum in the chosen energy range (WIMP particles have more energy), making the detection more favourable for low masses
- Including a <1 form factor that depends on the transferred momentum has a clear impact, especially for higher masses

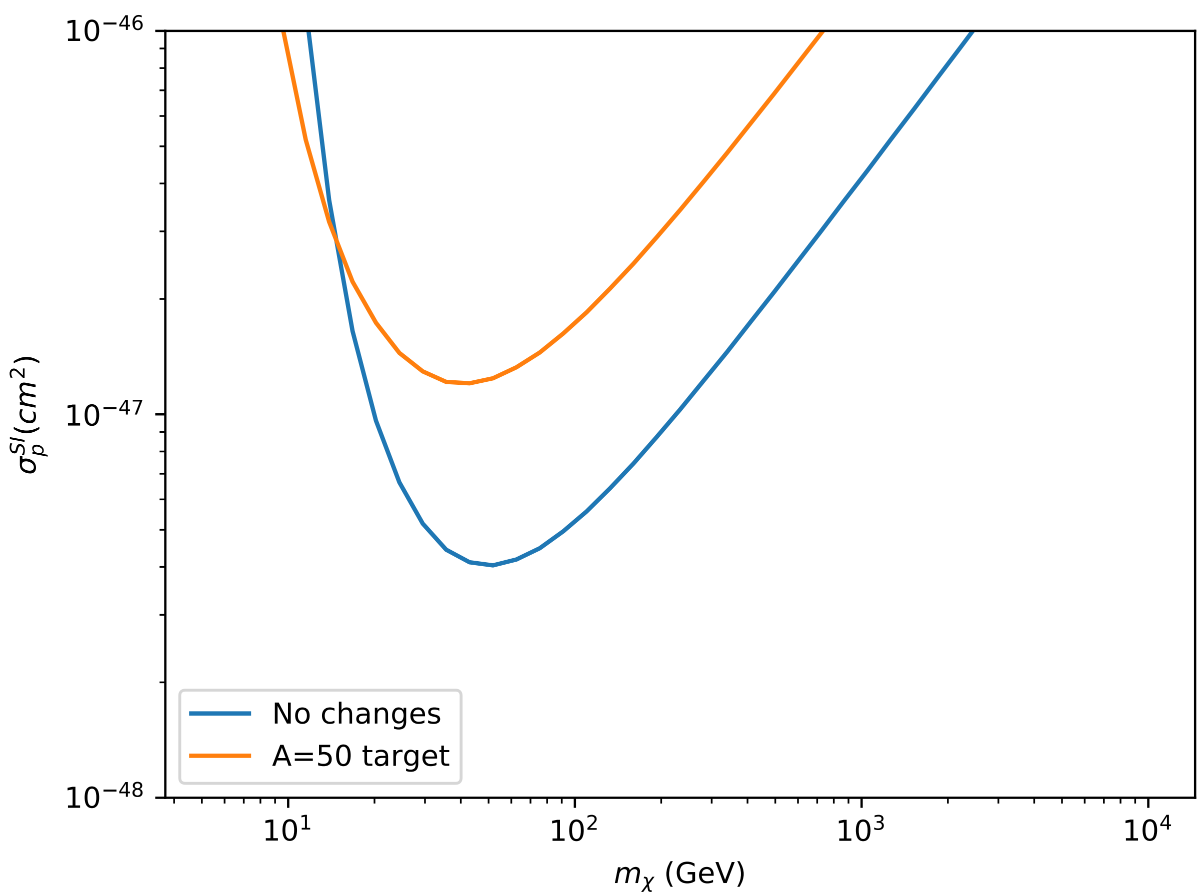


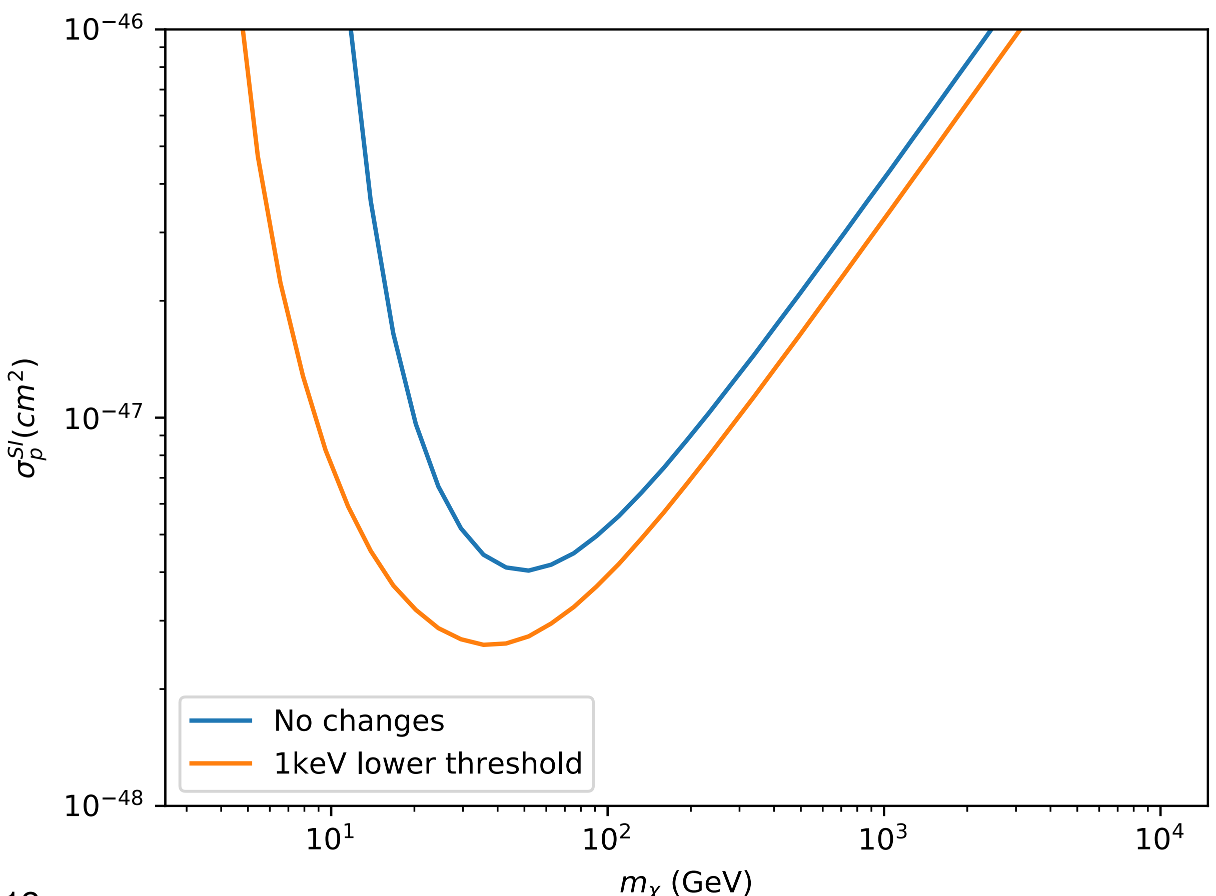


Other interesting effects

Feel free to try these in your homework

- With a lighter target we gain some sensitivity at low masses, but lose on the coherence factor (A^2)
- A low energy threshold has a big impact on these experiments, particularly (but not only) at low masses





Bibliography

- “A History of Dark Matter”, by Gianfranco Bertone and Dan Hooper, Rev. Mod. Phys. 90, 45002 (2018), https://arxiv.org/abs/1605.04909, https://doi.org/10.1103/RevModPhys.90.045002
- *“Particle Dark Matter”*, editor Gianfranco Bertone, Cambridge University Press, August 2010, ISBN: 9780511770739, https://doi.org/10.1017/CBO9780511770739.
- *“Review of mathematics, numerical factor and corrections for dark matter experiments based on elastic nuclear recoils”*, by J.D. Lewin and P.F. Smith, Astroparticle Physics 6 (1996)
- “A Unified Approach to the Classical Statistical Analysis of Small Signals”, by G.J. Feldman and R.D. Cousins, PRD 57 (1998) 3873, arXiv:physics/9711021
- “*TASI lectures on dark matter models and direct detection*”, by Tongyan Lin, arXiv:1904.07915.
- “Lectures on Dark Matter Physics”, Mariangela Lisanti, http://indico.ictp.it/event/7624/session/16/contribution/72/material/0/0.pdf
- “Direct Dark Matter Detection”, Henrique Araújo, IDPSC School 2021, https://indico.lip.pt/event/643/
- “Dark Matter Searches”, Laura Baudis, BND School 2009, http://bnd-graduateschool.org/2009/?Lecture_Notes